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Differential quadrature method for vibration analysis of beams on viscoelastic foundations

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Abstract: This paper investigates transverse vibrations of finite Euler-Bernoulli beams resting on viscoelastic foundations. It studies natural frequencies and dynamic response of an elastic beam subjected to a harmonic load. The differential quadrature methods(DQ) are applied directly to the governing equations of the free and the forced vibrations that are two partial differential equations. Under the simple supported boundary condition, the natural frequencies of the transverse vibrations are calculated, and compared with the results of the complex modal analysis method. The natural frequencies and dynamic response are numerically studied. The numerical results obtained with the DQ are in good agreement with that of the complex mode analysis methods for the first seven orders, but with the growth of the orders, the small quantitative differences between them increase. Numerical results also illustrate that the beam vibrates with same frequency of external load under harmonic motion, after a short transient response.

Key words: viscoelastic foundation; natural frequency; dynamic response; differential quadrature methods

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1 Introduction

Vibration analysis of elastic Euler-Bernoulli beams resting on different types of foundations is of great interest in the area of transportation and civil engineering. The vibration problem of elastic beams resting on viscoelastic foundation displays viscous characters, and the solution becomes difficult. Most of literatures report free vibration analysis of beam on Pasternak foundation with approximate solutions. In most of the published researches on the topic of a finite beam resting on a foundation, the foundation is assumed as linear elastic one, and the problems are mainly studied by approximate methods such as finite element method[1], transfer matrix method[2], Rayleigh-Ritz method[3], differential quadrature methods[4], and Laplace transform technique[5].

It is worth noticing that all studies mentioned above are about elastic foundation, without accounting for any damping factor in foundation. But in fact, the effects of damping factors are an important research topic for vibration of elastic beams on foundation. In recent years, researchers began to study vibration of elastic beams

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resting on viscoelastic three parameters foundation. Free transverse vibration of beams on viscoelastic Winkler and Pasternak foundation are studied with complex mode analysis methods[6-8], but most of the published works on this specific problem are focused on dynamic response of such a system subjected to a moving load[9-10].

The differential quadrature (DQ) method is a powerful tool for dealing with dynamical problems. It was introduced for structural analysis by Bert[11], and since then it has been successfully employed for the analysis of vibration of beams[12]. The dynamic response problem of elastic beams lying on viscoelastic foundation displays viscous characters, and the solution becomes difficult. To the author's best knowledge, there were no literatures on the dynamic response of beams on viscoelastic Pasternak foundation excited by harmonic loads. In the present work, using the DQ method, the natural frequencies and dynamic responses are analyzed. The numerical results obtained by the DQ are compared with the complex modal analysis methods.

The present paper is organized as follows. Section 2 establishes the governing equation for the transverse free vibration of a finite elastic Euler-Bernoulli beam on viscoelastic foundation. Section 3 develops the DQ schemes to discretize the governing equation. Section 4 presents numerical examples for investigations on the effects of the relevant beam and foundation parmeters on the natural frequencies and the dynamic responses. Section 5 ends the paper with the concluding remarks.

2 Equation of motion

For a finite elastic Euler-Bernoulli beam with a length l resting on viscoelastic foundation, the equation of free motion can be written as[12]

$$\rho AY_{,tt} + EIY_{,xxxx} + k_1 y - k_2 y_{,xx} + cy_{,t} = 0, \tag{1}$$

where A, E, I, ρ, y, x and t are cross-sectional area of the beam, modulus of elasticity, cross-sectional moment of inertia, beam material density, beam transverse deflection, axial coordinate of the beam and time, respectively. k_1, k_2 and c are linear parameter, shearing parameter and viscous damping coefficient of the foundation.

Introducing the dimensionless variables and parameters as follows:

$$y \leftrightarrow \frac{y}{l}, t \leftrightarrow \frac{t}{l}\sqrt{\frac{E}{\rho}}, x \leftrightarrow \frac{x}{l}, k_f \leftrightarrow \frac{1}{l}\sqrt{\frac{I}{A}}, k_f \leftrightarrow \frac{k_2 l^2}{EA}, c \leftrightarrow \frac{c}{A}\sqrt{\frac{l^2}{\rho E}}. \tag{2}$$

Substitution of equation (2) into equation (1) yields the dimensionless governing equations

$$y_{,tt} + k_f^2 y_{,xxxx} + k_1 y - k_2 y_{,xx} + cy_{,t} = 0. \tag{3}$$

The dimensionless pinned-pinned boundary conditions of the beam can be expressed by

$$y(0,t) = y(1,t) = 0, y''(0,t) = y''(1,t) = 0. \tag{4}$$

3 Differential quadrature method

The method of DQ is employed to solve the problem. This method requires to discretize the domain of the problem into N points. Then the derivatives at any point are approximated by a weighted linear summation of all the functional values along the discretized domain.

Consider the computational domain $0 \leqslant x \leqslant 1$ of the beam. The number of sampling point is N in the x direction. Introduce N sampling point as [13]

$$x_0, x_N = 1, x_i = \frac{1}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right](i - 2, 3, \cdots, N-1). \tag{5}$$

The quadrature rules for the derivatives of a function at the sampling points yield

$$y_{,xx}(x,t) = \sum_{j=1}^{N} A_{ij}^{(2)} y(x_j,t), \quad y_{,xxxx}(x_i,t) = \sum_{j=1}^{N} A_{ij}^{(4)} y(x_j,t). \tag{6}$$

Substitution of equation (7) into equations (4) yields

$$\ddot{y}_i + c\dot{y}_i + \sum_{k=1}^{N} (k_f^2 A_{ik}^{(4)} - k_2 A_{ik}^{(2)}) y_k + k_1 y_i = 0, \tag{7}$$

where

$$y_i(t) = y(x_i, t), \tag{8}$$

under simply supported boundary conditions

$$y_1 = y_N = 0, \sum_{k=1}^{N} A_{1k}^{(2)} y_k = \sum_{k=1}^{N} A_{Nk}^{(2)} y_k = 0, \tag{9}$$

where the weight coefficient is defined by

$$A_{ik}^{(r)} = \begin{cases} \prod_{\mu=1,\mu\neq i}^{N} (x_i - x_\mu) / [(x_i - x_k) \prod_{\mu=1,\mu\neq k}^{N} (x_k - x_\mu)] & i,k = 1,2,\cdots,N, i = \neq k \\ \sum_{\mu=1,\mu\neq i}^{N} \frac{1}{x_i - x_\mu} & i = 1,2,\cdots N, i = k \end{cases} \tag{10}$$

and in case of $r = 2,3,\cdots, N-1$

$$A_{ik}^{(r)} = \left\{ r\left(A_{ii}^{(r-1)} A_{ik}^{(1)} \frac{A_{ik}^{(r-1)}}{x_i - x_k} \right) = - \sum_{\mu=1,\mu\neq i}^{N} A_i^{(r)} \sum_{\mu=1,\mu\neq i}^{N} (i = 1,2,\cdots,N, i = k). \right. \tag{11}$$

To overcome difficulties in the implementation of the boundary conditions, the modified weighting coefficient matrix are introduced as

$$[\tilde{A}^{(2)}] = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ A_{21}^{(2)} & A_{22}^{(2)} & \cdots & A_{2N}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ A_{(N-1)1}^{(2)} & A_{(N-1)2}^{(2)} & \cdots & A_{(N-1)N}^{(2)} \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{12}$$

$$[\tilde{A}^{(r)}] = [A^{(1)}][\tilde{A}^{(r-1)}] \qquad (r = 3,4,\cdots). \tag{13}$$

Substitution of matrix (12) and (12) into equation (12), one obtain

$$\ddot{y}_i + c\dot{y}_i + \sum_{k=1}^{N} (k_f^2 \tilde{A}_{ik}^{(4)} - k_2 \tilde{A}_{ik}^{(2)}) y_k + k_1 y_i = 0. \tag{14}$$

4 Numerical results

In order to testify the result of the complex modal analysis methods[7], consider a same finite elastic Euler-Bernoulli beam resting on viscoelastic foundation, the data of physical and geometrical properties of the beam and foundation come from the model beam[14]. Substitution the data into equation (2) yields the dimensionless value: $k_f = 0.0035$, $k_1 = 7.02$, $k_2 = 0.367$, $c = 0.10$. Choose sampling points $N = 19$.

4.1 Natural frequencies

For the free motion of the beam resting on viscoelastic foundation, Figure 1 and Figure 2 show that deflections of mid-point of the beam decrease with time increase. In Table 1, the first ten natural frequencies of the model beam are given. It shows that the first seven natural frequencies of the beam with DQ are in good agreement with the result of complex mode analysis methods (CMA)[7], but from the eighth order, there is a very small quantitative difference between them. Compared with the complex mode method, the accuracy of the result of the DQ method are concentrated in the first few orders, and after seven orders, with the growth of the order, the error increases.

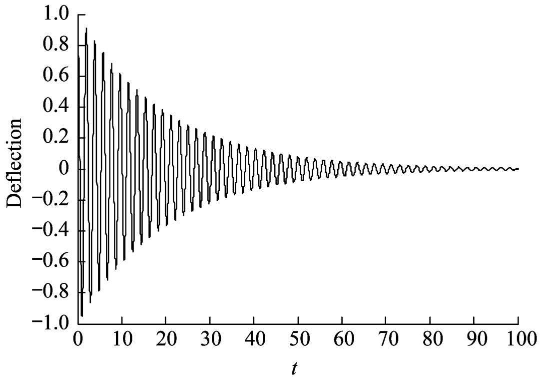

Figure 1　Time history

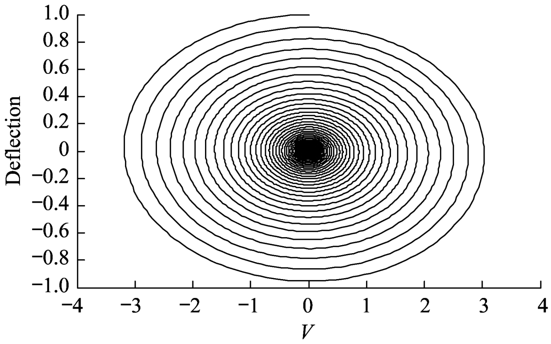

Figure 2　Phase trajectory

Table 1　The natural frequencies for the first ten orders with DQ via CMA

ω_n	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9
DQ	3.2620	4.6395	6.3019	8.0794	9.9155	11.7882	13.6882	15.6116	17.5568
CMA	3.2620	4.6395	6.3019	8.0794	9.9155	11.7882	13.6881	15.6159	17.5073

4.2　Dynamic response

Consider a ground base beam of a steel structure industrial building, which has two simply supported ends, bearing uniformly distributed harmonic loads. The upper loads affect the dynamic response of the ground beam, which is important for the safety of the structure.

The governing differential equation of motion for the model beam subjected to a harmonic load can be written as

$$y_{,tt} + k_f^2 y_{,xxxx} + k_1 y - k_2 y_{,xx} + c y_{,t} = F_0 \cos(\omega t), \tag{15}$$

where F_0 and are dimensionless load and frequency.

Taking the dimensionless value of load ($F_0 = 15$) and frequency ($\omega = 4$), Figure 3 illustrates the dynamic response of the midpoint of the beam. The time history and phase trajectory diagrams show that the beam vibrates with same frequency of external load under harmonic motion, after a short transient response.

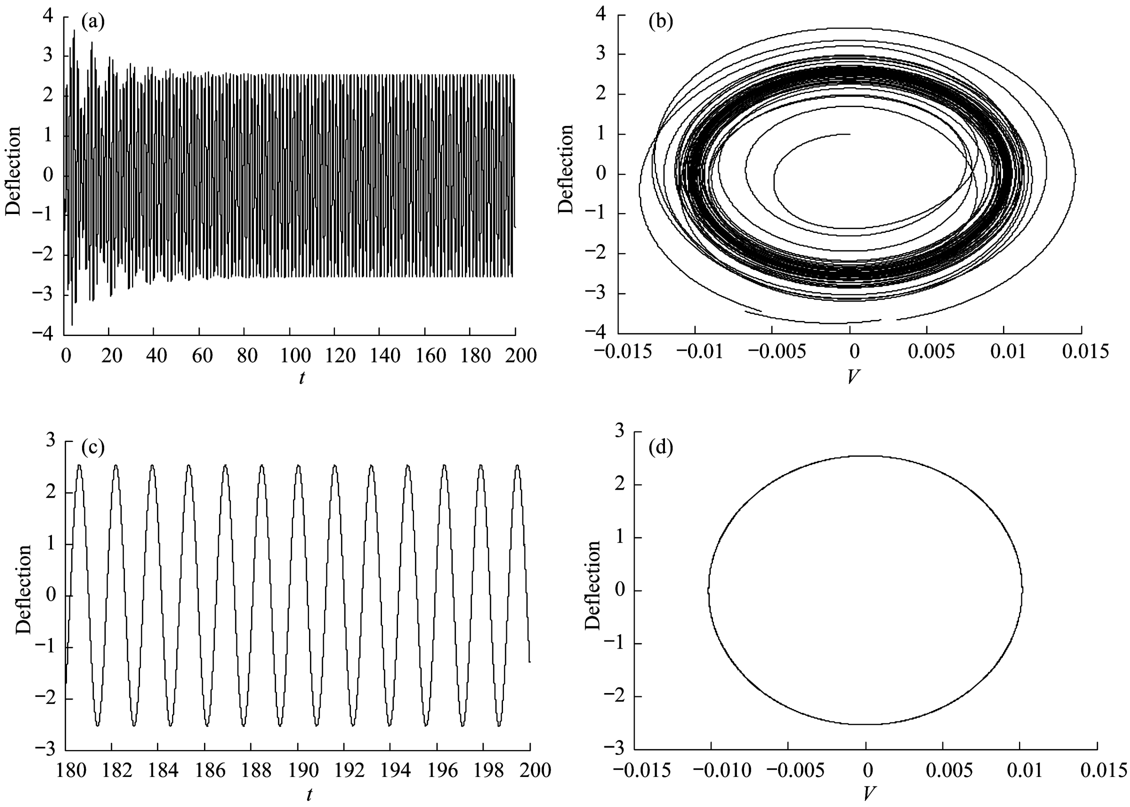

Figure 3 Dynamic response of a beam midpoint beneath the load ($\omega = 4$): (a) Time history; (b) Phase rajectory; (c) Partial enlargement of (a); (d) Partial enlargement of (b).

5 Conclusions

This paper investigates transverse vibrations of finite Euler-Bernoulli beams resting on three-parameter viscoelastic foundations. It studies natural frequencies and dynamic response of an elastic beam subjected to a harmonic load. The differential quadrature methods are applied directly to the governing equations of the free and the forced vibrations which are two partial differential equations. Under the simply supported boundary condition, the natural frequencies of the transverse vibrations are compared with the results of the complex modal analysis method. The natural frequencies and dynamic response are numerically studied. The numerical results obtained with the differential quadrature are in good agreement with that of the complex mode methods for the first seven orders, but with the growth of the orders, the small quantitative differences between them increase. Numerical results also illustrate that the beam vibrates with same frequency of external load under harmonic motion, after a short transient response.

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黏弹性地基梁振动的微分求积法分析

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摘　要: 探讨了黏弹性地基上有限长 Euler-Bernoulli 梁的横向振动. 主要研究梁的固有频率和简谐均布荷载作用下的动力响应. 将微分求积方法(DQ)直接应用于自由与受迫振动控制方程中. 在简支边界条件下,得到横向自由振动的固有频率,并与复模态分析方法的结果进行比较. 数值结果表明 DQ 与复模态分析方法得到的前七阶频率值高度吻合,但随着阶数的增长,两种方法数值间的微小差异值增大. 数值结果还表明, 在均布简谐荷载作用下,经过短暂的瞬态响应后,梁的振动频率与外部荷载振动频率一致.

关键词: 黏弹性地基; 固有频率; 动力响应; 微分求积法

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