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Periodic solutions of a nonautonomous predator-prey model with a constant refuge

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Abstract: We consider the nonautonomous predator-prey model with a constant refuge. Some new and interesting sufficient conditions are obtained for the global existence of the positive periodic solutions of such a model. Our method is based on Mawhin's coincidence degree and novel estimation techniques for the a priori bounds of unknown solutions to $Lx = \lambda Nx$.

Key words: predator-prey model; refuge; periodic solution.

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1 Introduction

In the past few years, the quantitive theory of dynamic system has been well developed and it has been applied to various fields (see e.g. [1–17] and references therein). In particular, the biomathematical dynamics has been quickly developed. The classical Gauss type predator-prey models can be expressed as follows,

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} &=& g(x) - \varphi(x,y)y \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} &=& c\varphi(x,y)y - \mathrm{d}y \end{cases}, \tag{1}$$

where $x = x(t)$ and $y = y(t)$ represent predator and prey densities at time t, respectively, and $g(x)$ is the growth rate of the prey population in the absence of predation. The positive constant c describes the efficiency of the predator in converting the consumed prey into the predator offspring, d denotes the food independent predator mortality rate, and $\varphi(x,y)$ denotes the instantaneous rate of prey depletion per predation. Recently, some authors[18–21] managed to incorporate a constant prey refuge m into the predator-prey models of the form

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} &=& ax\Big(1-\dfrac{x}{k}\Big) - \dfrac{\beta(x-m)y}{1+\alpha(x-m)} \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} &=& \Big[-d+\dfrac{c\beta(x-m)}{1+\alpha(x-m)}\Big]y \end{cases}. \tag{2}$$

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Most of the models are assumed to be autonomous, that is, all biological or environmental parameters are assumed to be constants in time. However, many biological and environmental parameters do vary in time (e.g., naturally subject to seasonal fluctuations, mating habits). Taking these into consideration, a model should be non-autonomous. In particular, many authors assumed that the parameters are periodic or almost periodic for this reason. In this paper, we consider the nonautonomous predator-prey model with a constant refuge m,

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)\Big[a(t)-b(t)x(t)\Big]-\dfrac{\beta(t)(x(t)-m)y(t)}{1+\alpha(x(t)-m)} \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = \Big[-d(t)+\dfrac{c(t)\beta(t)(x(t)-m)}{1+\alpha(x(t)-m)}\Big]y(t) \end{cases}, \tag{3}$$

where α and m are positive constants, m is a constant number of prey using refuges, which protects m of prey from predation, $a(t)$ represents the intrinsic growth rate at time t, $d(t)$ represents the death rate of the predator at time t, $c(t)$ represents the conversion factor denoting the number of newly born predators for each captured prey at time t. The term $\dfrac{\beta x}{1+\alpha x}$ denotes the functional response of the predator, which is termed as Holling type II response function (see e.g. [22]). Adopt the following symbol throughout this paper,

$$\bar{f}=\frac{1}{\omega}\int_0^{\omega} f(t)\mathrm{d}t,$$

where f is a continuous ω-periodic function.

2 Existence of positive periodic solutions

Firstly, we introduce some symbols and lemmas. Let X, Y be normed vector spaces, $L:\mathrm{Dom}L\subset X\to Y$ be a linear mapping, and $N: X\to Y$ be a continuous mapping. The mapping L is called a Fredholm mapping of index zero if dimKer$L=\mathrm{codim}\,\mathrm{Im}L<+\infty$ and ImL is closed in Y. If L is a Fredholm mapping of index zero, there exist continuous projectors $P: X\to X$ and $Q: Y\to Y$ such that Im$P=\mathrm{Ker}L$ and Ker$Q=\mathrm{Im}L=\mathrm{Im}(I-Q)$. It follows that $L|\mathrm{dom}L\cap\mathrm{Ker}P:(I-P)X\to\mathrm{Im}L$ is invertible. Let the inverse of that map be denoted by K_p. If Ω is an open bounded subset of X, then the mapping N will be said to be L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_p(I-Q)N:\overline{\Omega}\to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J:\mathrm{Im}Q\to\mathrm{Ker}L$.

Lemma 1 [23] Let $\Omega\subset X$ be an open bounded set. Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Assume that

(a) for each $\lambda\in(0,1)$, $x\in\partial\Omega\cap\mathrm{Dom}L$, $Lx\neq\lambda Nx$;

(b) for each $x\in\partial\Omega\cap\mathrm{Ker}L$, $QNx\neq 0$;

(c) $\deg\{JQN,\ \Omega\cap\mathrm{Ker}L,\ 0\}\neq 0$.

Then $Lx=Nx$ has at least one solution in $\overline{\Omega}\cap\mathrm{Dom}L$.

Lemma 2 Suppose that

(H_1) $\overline{d}<\dfrac{\overline{c\beta}}{\alpha}$,

(H_2) $\dfrac{\overline{d}+m(\overline{c\beta}-\alpha\overline{d})}{\overline{c\beta}-\alpha\overline{d}}<\dfrac{\overline{a}}{\overline{b}}$.

Then the algebraic equations

$$\begin{cases} \overline{a}-\overline{b}v_1-\dfrac{1}{\omega}\displaystyle\int_0^{\omega}\dfrac{\beta(t)(v_1-m)v_2}{v_1+\alpha v_1(v_1-m)}\mathrm{d}t=0 \\ -\overline{d}+\dfrac{1}{\omega}\displaystyle\int_0^{\omega}\dfrac{c(t)\beta(t)(v_1-m)}{1+\alpha(v_1-m)}\mathrm{d}t=0 \end{cases}, \tag{4}$$

have a unique solution $(v_1^*,v_2^*)^T\in\mathbb{R}^2$ with $v_i^*>0, i=1,2$.

Proof Consider the function

$$f(u)=\overline{d}-\frac{1}{\omega}\int_0^{\omega}\frac{c(t)\beta(t)}{u+\alpha}\mathrm{d}t.$$

From (H_1) , we can see that

$$f(0)=\overline{d}-\frac{\overline{c\beta}}{\alpha}<0,\quad \lim_{u\to+\infty}f(u)=\overline{d}>0.$$

then from the zero point theorem and the monotonicity of $f(u)$, it follows that there exists a unique $u^*>0$ such that $f(u^*)=0$.

Now, we can see that $u^*=\dfrac{1}{v_1^*-m}$ is a solution of the second equation of system (4). Substitute $u^*=\dfrac{1}{v_1^*-m}$ into the equation of (1) and simplify, we have

$$v_1^*=\frac{1}{u^*}+m>0,$$

$$v_2^*=(-\overline{b}(v_1^*)^2+\overline{a}v_1^*)\frac{u^*+\alpha}{\overline{\beta}}>0.$$

From (H_2) , noting $0<m<v_1^*<\dfrac{\overline{a}}{\overline{b}}$, we get that $g(v_1^*)=-\overline{b}(v_1^*)^2+\overline{a}v_1^*>0$. The proof is complete.

Theorem 1 In addition to (H_1) and (H_2), suppose that

(H_3) $\dfrac{\overline{d}}{\overline{c\beta}\exp\{2\overline{a}\omega\}}-\dfrac{\overline{a}\exp\{4\overline{a}\omega\}}{\overline{b}}>0,$

(H_4) $\dfrac{\overline{c\beta}\exp\{2\overline{a}\omega\}}{\overline{d}-m\overline{c\beta}\exp\{2\overline{a}\omega\}}+\alpha>0.$

Then the system (3) has at least two positive ω-periodic solutions.

Proof Make the change of variables

$$x(t)=\exp\{u_1(t)\},\quad y(t)=\exp\{u_2(t)\}.$$

The system (3) can be rewritten as

$$\begin{cases} \dot{u}_1(t) = a(t)-b(t)\exp\{u_1(t)\}-\dfrac{\beta(t)(\exp\{u_1(t)\}-m)\exp\{u_2(t)-u_1(t)\}}{1+\alpha(\exp\{u_1(t)\}-m)} \\ \dot{u}_2(t) = -d(t)+\dfrac{c(t)\beta(t)(\exp\{u_1(t)\}-m)}{1+\alpha(\exp\{u_1(t)\}-m)} \end{cases}. \tag{5}$$

Take

$$X=Y=\{x=(u_1,u_2)^T\in C(\mathbb{R},\mathbb{R}^2\mid x(t+\omega)=x(t)\},$$

and define

$$\|x\|=\max_{t\in[0,\omega]}|u_1(t)|+\max_{t\in[0,\omega]}|u_2(t)|,\quad x=(u_1,u_2)^T\in X \text{ or } Y.$$

Here $|\cdot|$ denotes the Euclidean norm. Then X and Y are Banach spaces with the norm $\|\cdot\|$. For any $x=(u_1,u_2)^T\in X$, by means of the periodicity assumption, we can easily check that

$$a(t)-b(t)\exp\{u_1(t)\}-\frac{\beta(t)(\exp\{u_1(t)\}-m)\exp\{u_2(t)-u_1(t)\}}{1+\alpha(\exp\{u_1(t)\}-m)}:=\Delta_1(u,t)\in C(\mathbb{R},\mathbb{R}),$$

$$-d(t)+\frac{c(t)\beta(t)(\exp\{u_1(t)\}-m)}{1+\alpha(\exp\{u_1(t)\}-m)}:=\Delta_2(u,t)\in C(\mathbb{R},\mathbb{R})$$

are ω-periodic.

And define the operators $L:\mathrm{DomL}\subset \mathrm{X}\to \mathrm{Z}$ and $N:X\to Z$ as follows,

$$X\ni x(t)\to (Lx)(t)=\frac{\mathrm{d}x(t)}{\mathrm{d}t}\in Z,$$

$$X\ni x(t)\to (Nx)(t)=\big((Nu)_1(t),(Nu)_2(t)\big)^T\in Z,$$

where

$$(Nu)_i(t)=\Delta_i(u,t),\ i=1,2.$$

Define, respectively, the projectors $P:X\to X$ and $Q:Z\to Z$ by

$$Px(t)=\frac{1}{\omega}\int_0^{\omega}x(t)\mathrm{d}t,\ \ Qz(t)=\frac{1}{\omega}\int_0^{\omega}z(t)\mathrm{d}t,\ x\in X,\ z\in Z.$$

It can be seen that the domain of L in X is actually the whole space, and

$$\mathrm{Ker}L=\{x(t)\in X|Lx(t)=0,\ \text{i.e. } \dot{x}(t)=0\}=\mathbb{R}^2,$$

$$\mathrm{Im}L=\{z(t)\in Z|\int_0^{\omega}z(t)\mathrm{d}t=0\}\ \text{is closed in } Z.$$

Moreover, P,Q are continuous operators such that

$$\mathrm{Im}P=\mathbb{R}^2=\mathrm{Ker}L,\qquad \mathrm{Im}L=\mathrm{Ker}Q=\mathrm{Im}(I-Q)$$

and

$$\mathrm{dimKer}L=\mathrm{codimIm}L=2<+\infty.$$

It follows that L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_P:\mathrm{Im}L\to \mathrm{Dom}L\cap\mathrm{Ker}P$ exists, which is given by

$$K_P(y)=\int_0^t y(s)\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^t y(s)\mathrm{d}s\mathrm{d}t.$$

Then $QN:X\to Z$ and $K_P(I-Q)N:X\to X$ are defined by

$$QNx=\left(\frac{1}{\omega}\int_0^{\omega}\Delta_1(u,t)\mathrm{d}t,\frac{1}{\omega}\int_0^{\omega}\Delta_2(u,t)\mathrm{d}t\right)^T,$$

$$K_P(I-Q)Nu=\int_0^t\Delta_k(u,s)\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^t\Delta_k(u,s)\mathrm{d}s\mathrm{d}t-\left(\frac{t}{\omega}-\frac{1}{2}\right)\int_0^{\omega}\Delta_k(u,s)\mathrm{d}s,\ k=1,2.$$

Clearly, QN and $K_p(I-Q)N$ are continuous. By using the Arzelà-Ascoli Theorem, it is not difficult to prove that $K_p(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega\subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Therefore, N is

L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$. Now, we shall search for an appropriate open bounded subset in order to apply Lemma 1.

Corresponding to the operator equation $Lu = \lambda Nu$, $\lambda \in (0,1)$, we have

$$\dot{u}_1(t) = \lambda\Big[a(t) - b(t)\exp\{u_1(t)\} - \frac{\beta(t)(\exp\{u_1(t)\} - m)\exp\{u_2(t) - u_1(t)\}}{1 + \alpha(\exp\{u_1(t)\} - m)}\Big], \tag{6}$$

$$\dot{u}_2(t) = \lambda\Big[-d(t) + \frac{c(t)\beta(t)(\exp\{u_1(t)\} - m)}{1 + \alpha(\exp\{u_1(t)\} - m)}\Big]. \tag{7}$$

Suppose that $u = (u_1(t), u_2(t))^T \in X$ is a solution of (6) and (7) for a certain $\lambda \in (0,1)$. Integrating (6), (7) over the interval $[0,\omega]$, we obtain

$$\int_0^\omega \Big[b(t)\exp\{u_1(t)\} + \frac{\beta(t)(\exp\{u_1(t)\} - m)\}\exp\{u_2(t) - u_1(t)\}}{1 + \alpha(\exp\{u_1(t)\} - m)}\Big]\mathrm{d}t = \overline{a}\omega, \tag{8}$$

$$\int_0^\omega \frac{c(t)\beta(t)(\exp\{u_1(t)\} - m)}{1 + \alpha(\exp\{u_1(t)\} - m)}\mathrm{d}t = \overline{d}\omega. \tag{9}$$

It follows from (6) and (8) that

$$\int_0^\omega |\dot{u}_1(t)|\mathrm{d}t = \lambda\int_0^\omega \Big|a(t) - b(t)\exp\{u_1(t)\} - \frac{\beta(t)(\exp\{u_1(t)\} - m)\exp\{u_2(t) - u_1(t)\}}{1 + \alpha(\exp\{u_1(t)\} - m)}\Big|\mathrm{d}t <$$
$$\int_0^\omega \Big[a(t)\mathrm{d}t + \int_0^\omega b(t)\exp\{u_1(t)\} + \frac{\beta(t)(\exp\{u_1(t)\} - m)\exp\{u_2(t) - u_1(t)\}}{1 + \alpha(\exp\{u_1(t)\} - m)}\Big]\mathrm{d}t =$$
$$\int_0^\omega a(t)\mathrm{d}t + \overline{a}\omega = 2\overline{a}\omega,$$

that is,

$$\int_0^\omega |\dot{u}_1(t)|\mathrm{d}t < 2\overline{a}\omega. \tag{10}$$

Similarly, it follows from (7) and (9) that

$$\int_0^\omega |\dot{u}_2(t)|\mathrm{d}t < 2\overline{d}\omega. \tag{11}$$

Since $(u_1(t), u_2(t))^T \in X$, there exist $\xi_i, \eta_i \in [0,\omega]$ such that

$$u_i(\xi_i) = \min_{t\in[0,\omega]} u_i(t),\ \ u_i(\eta_i) = \max_{t\in[0,\omega]} u_i(t),\ \ i = 1, 2. \tag{12}$$

It follows from (8) that

$$\overline{a}\omega \geqslant \int_0^\omega b(t)\exp\{u_1(t)\}\mathrm{d}t,$$

that is

$$\overline{a} \geqslant \overline{b}\exp\{u_1(\xi_1)\},$$

which implies

$$u_1(\xi_1) \leqslant \ln\frac{\overline{a}}{\overline{b}}.$$

This, combined with (10), gives

$$u_1(t) \leqslant u_1(\xi_1) + \int_0^{\omega} |\dot{u}_1(t)|\mathrm{d}t \leqslant \ln\frac{\overline{a}}{\overline{b}} + 2\overline{a}\omega \triangleq H_{11}. \tag{13}$$

Similarly, it follows from (9) that

$$\overline{d}\omega = \int_0^{\omega} \frac{c(t)\beta(t)(\exp\{u_1(t)\} - m)}{1 + \alpha(\exp\{u_1(t)\} - m)}\mathrm{d}t,$$

which implies

$$\overline{d}\omega \leqslant \int_0^{\omega} c(t)\beta(t)\exp\{u_1(t)\}\mathrm{d}t,$$

that is

$$\overline{d} \leqslant \overline{c\beta}\exp\{u_1(\eta_1)\}.$$

So we have

$$u_1(\eta_1) \geqslant \ln\frac{\overline{d}}{\overline{c\beta}}.$$

This, combined with (10), gives

$$u_1(t) \quad \geqslant \quad u_1(\eta_1) - \int_0^{\omega} |\dot{u}_1(t)|\mathrm{d}t \geqslant \ln\frac{\overline{d}}{\overline{c\beta}} - 2\overline{a}\omega \triangleq H_{12}. \tag{14}$$

It follows from (13) and (14)that

$$\max_{t\in[0,\omega]} |u_1(t)| < \max\{|H_{11}|, |H_{12}|\} \triangleq H_1. \tag{15}$$

Multiplying (6) by $\exp\{u_1(t)\}$ and integrating over $[0,\omega]$,

$$\int_0^{\omega} \Big[b(t)\exp\{2u_1(t)\} + \frac{\beta(t)(\exp\{u_1(t)\} - m)\exp\{u_2(t)\}}{1 + \alpha(\exp\{u_1(t)\} - m)}\Big]\mathrm{d}t = \int_0^{\omega} a\exp\{u_1(t)\}\mathrm{d}t,$$

then,we have

$$\int_0^{\omega} \Big[b(t)\exp\{2u_1(t)\} + \frac{\beta(t)(\exp\{u_1(t)\} - m)\}\exp\{u_2(t)\}}{\alpha(\exp\{u_1(t)\} - m)}\Big]\mathrm{d}t \geqslant \int_0^{\omega} a\exp\{u_1(t)\}\mathrm{d}t.$$

Which implies

$$\int_0^{\omega} \Big[b(t)\exp\{2u_1(t)\} + \frac{\beta(t)}{\alpha}\exp\{u_2(t)\}\Big]\mathrm{d}t \geqslant \int_0^{\omega} a\exp\{u_1(t)\}\mathrm{d}t. \tag{16}$$

It follows from (13), (14)and (H_3) that

$$u_2(\eta_2) \geqslant \ln\Big[\frac{\overline{a}\alpha}{\overline{\beta}}(\frac{\overline{d}}{\overline{c\beta}\exp\{2\overline{a}\omega\}} - \frac{\overline{a}\exp\{4\overline{a}\omega\}}{\overline{b}})\Big].$$

This, combined with (11), gives

$$u_2(t) \quad \geqslant \quad u_2(\eta_2) - \textstyle\int_0^{\omega} |\dot{u}_2(t)|\mathrm{d}t > \ln\Big[\frac{\overline{a}\alpha}{\overline{\beta}}(\frac{\overline{d}}{\overline{c\beta}\exp\{2\overline{a}\omega\}} - \frac{\overline{a}\exp\{4\overline{a}\omega\}}{\overline{b}})\Big] - 2\overline{d}\omega \triangleq H_{21}. \tag{17}$$

Similarly, it follows from (8) that

$$\int_0^{\omega} \frac{\beta(t)(\exp\{u_1(t)\} - m)\exp\{u_2(t) - u_1(t)\}}{1 + \alpha(\exp\{u_1(t)\} - m)}\mathrm{d}t \leqslant \overline{a}\omega.$$

Which implies

$$\int_0^\omega \frac{\beta(t)\exp\{u_2(t)\}}{\frac{1}{\exp\{u_1(t)\}-m}+\alpha}\mathrm{d}t \leqslant \int_0^\omega a\exp\{u_1(t)\}\mathrm{d}t.$$

It follows from (13), (14)and (H_4) that

$$u_2(\xi_2) \leqslant \ln\Big[\frac{\overline{a}^2\exp\{2\overline{a}\omega\}}{\overline{b}\times\overline{\beta}}\big(\frac{\overline{c\beta}\exp\{2\overline{a}\omega\}}{\overline{d}-m\overline{c\beta}\exp\{2\overline{a}\omega\}}+\alpha\big)\Big].$$

This, combined with (11), gives

$$u_2(t) \leqslant u_2(\xi_2)+\int_0^\omega |\dot{u}_2(t)|\mathrm{d}t < \ln\Big[\frac{\overline{a}^2\exp\{2\overline{a}\omega\}}{\overline{b}\times\overline{\beta}}\big(\frac{\overline{c\beta}\exp\{2\overline{a}\omega\}}{\overline{d}-m\overline{c\beta}\exp\{2\overline{a}\omega\}}+\alpha\big)\Big]+2\overline{d}\omega \triangleq H_{22}. \quad (18)$$

It follows from (17) and (18) that

$$\max_{t\in[0,\omega]}|u_2(t)| < \max\{|H_{21}|,|H_{22}|\} \triangleq H_2. \quad (19)$$

Clearly, H_1, H_2 are independent of λ. Take $H = H_1 + H_2 + H_3$, where $H_3 > 0$ is taken sufficiently large such that $\|(v_1^*, v_2^*)\|_1 = \sum_{i=1}^{2}|v_i^*| < H_3$, where (v_1^*, v_2^*) is a solution of (4) with $v_1^* > 0, v_2^* > 0$.

Let $\Omega = \{(u_1, u_2)^T \in X\|(u_1, u_2)\| < H\}$, then it is clear that Ω verifies the requirement (a) of Lemma 1. Moreover, $QNx \neq 0$ for $x \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap \mathbb{R}^2$. A direct computation gives $\deg\{JQN,\ \Omega\cap\mathrm{Ker}L,\ 0\} \neq 0$. Here, J is taken as the identity mapping since $\mathrm{Im}Q = \mathrm{Ker}L$. So far we have proved that Ω satisfies all the assumptions in Lemma 1. Hence, system (3) has at least one ω-periodic solution. This completes the proof of Theorem 1.The proof is completed.

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具有避难所的非自治捕食系统的周期解

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摘　要：考虑了一类具有避难所的非自治捕食系统，得到了该模型存在正周期解的新颖充分条件. 所使用的方法是把 Mawhin 重合度理论和算子方程先验界的一些新估计技巧相结合.

关键词：捕食模型；避难所；周期解

(责任编辑：冯珍珍，包震宇)