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On the relationship between nonlinear and linear differential systems

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Abstract: In this article, we establish the relationship between the quadratic time-varying differential systems and the linear systems, giving the sufficient conditions for the quadratic systems to have the reflecting function in the form of fractional function. We use the obtained results to discuss the qualitative behavior of the solutions of the quadratic differential systems and the time-varying Kolmogrov equations.

Key words: reflecting function; periodic solution; time-varying system
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1 Introduction

To discuss the properties of the solutions of the differential system

$$x' = X(t, x) \tag{1}$$

is very important not only for the theory of ordinary differential equations but also for practical reason. When this system is an autonomous system, it attracts many scholars, and many excellent results are achieved[1-4]. However, we know little about the qualitative behavior of the solutions of non-autonomous systems. The book[5] shows that, a lot of biological models can be expressed by the time-varying differential system (1). Specially, when system (1) is a 2ω-periodic system, i.e., $X(t + 2\omega, x) = X(t, x)$ (ω is a positive constant), to study the behavior of the solutions of (1), we can use, as introduced in [6-7], the Poincaré mapping. But it is very difficult to find out the Poincaré mapping for many systems that are not integrable in finite terms. Mironenko[7] first established the theory of reflecting functions. Since then, a new method to establish the Poincaré mapping has been found.

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Consider the differential system (1) which has a continuously differentiable right-hand side and a general solution $\varphi(t;t_0,x_0)$. For such a system, the reflecting function is defined as $F(t,x) := \varphi(-t,t,x)$[7-8]. Therefore, for any solution $x(t)$ of (1), we have $F(t,x(t)) = x(-t)$.

If system (1) is 2ω-periodic with respect to t, then $T(x) := F(-\omega,x) = \varphi(\omega;-\omega,x)$ is the Poincaré mapping of (1) over the period $[-\omega,\omega]$. Thus, the solution $x = \varphi(t;-\omega,x_0)$ of (1) defined on $[-\omega,\omega]$ is 2ω-periodic if and only if x_0 is a fixed point of $T(x)$. The stability of this periodic solution is equivalent to the stability of the fixed point x_0.

A differentiable function $F(t,x)$ is a reflecting function of system (1) if and only if it is a solution of the Cauchy problem

$$F'_t + F'_x X(t,x) + X(-t,F) = 0, F(0,x) = x. \tag{2}$$

If $F(t,x) = F(t)x$ is the reflecting function of (1), then the matrix $F(t)$ is called the reflecting matrix. Thus, this matrix is the reflecting matrix of linear system $x' = P(t)x$, if and only if

$$F'(t) + F(t)P(t) + P(-t)F(t) = 0,\ F(0) = E.$$

If $F(t,x)$ is the reflecting function of (1), then it is also the reflecting function of the system

$$x' = X(t,x) + F_x^{-1}R(t,x) - R(-t,F(t,x)),$$

where $R(t,x)$ is an arbitrary vector function such that the solutions of the above systems are uniquely determined by their initial conditions. Therefore, all these 2ω-periodic systems have a common Poincaré mapping over the period $[-\omega,\omega]$, and the behavior of the periodic solutions of these systems are the same. So, to find out the reflecting function is very important for studying the qualitative behavior of solutions of the differential systems.

Many papers are also devoted to investigations of the qualitative behavior of the solutions of differential systems by help of reflecting functions. Mironenko[7−10] combined the theory of reflecting function with the integral manifold to discuss the symmetry and other geometric properties of solutions of (1), and obtained a lot of excellent conclusions. Alisevich[11] discussed the case when a linear system has triangular reflecting function. Musafirov[12] studied the case when a linear system has reflecting function that can be expressed as a product of three exponential matrices. Maiorovskaya[13] established the sufficient conditions under which the quadratic systems have linear reflecting functions. Zhou[14-16] discussed the structure of the reflecting functions of quadratic systems, and applied the conclusions to study the qualitative behavior of solutions of the differential systems.

In this paper, we will consider the time-varying quadratic differential system

$$\begin{cases} x' = a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 \\ y' = b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 \end{cases}, \tag{3}$$

where $a_i = a_i(t), b_i = b_i(t)\ (i = 1, 2, ..., 5)$ are continuously differentiable functions in R and there exists a unique solution for the initial value problem of (3). We will discuss the system (3) with the reflecting function in the form of

$$F(t,x,y) = \Big(\frac{\beta_1x + \beta_2y}{1 + \alpha_1x + \alpha_2y}, \frac{\gamma_1x + \gamma_2y}{1 + \alpha_1x + \alpha_2y}\Big)^T, \tag{4}$$

where $\alpha_i = \alpha_i(t), \beta_i = \beta_i(t), \gamma_i = \gamma_i(t)\,(i = 1, 2)$ are continuously differentiable functions in R and $\alpha_1^2 + \alpha_2^2 \neq 0$. We will give the sufficient conditions for the system (3) to have reflecting function (4), and establish the relationship between the system (3) and a linear system. After that, we use the obtained conclusions to study the behavior of the solutions of the system (3).

In the following , we set $a_i = a_i(t), \bar{a}_i = a_i(-t), F = F(t, x, y)$, etc.

2 Main result

Lemma 1 If the function (4) is the reflecting function of system (3), then

$$a_3 = b_4,\ a_4 = b_5,\ b_3 = a_5 = 0. \tag{5}$$

Proof Since the function (4) is the reflecting function of system (3), the equation (2) is held. Equating the coefficients of x^3, x^2y, xy^2, y^3 in this equation, we obtain

$$(b_3, a_3 - b_4, a_4 - b_5, a_5)\Delta_1 = 0,$$

$$(b_3, a_3 - b_4, a_4 - b_5, a_5)\Delta_2 = 0,$$

where $\Delta_1 = \beta_1\alpha_2 - \alpha_1\beta_2$, $\Delta_2 = \gamma_1\alpha_2 - \alpha_1\gamma_2$. If there is an identity of (5) not correct, then $\Delta_1 = \Delta_2 = 0$, i.e.,

$$\frac{\beta_1}{\beta_2} = \frac{\alpha_1}{\alpha_2} = \frac{\gamma_1}{\gamma_2}, \tag{6}$$

On the other hand, as $F(0, x, y) = (x, y)$, the identity (6) can not hold when $t = 0$. Thus, the conclusion of the present lemma is true.

Therefore, in the following, we only consider the system

$$\begin{cases} x' = a_1x + a_2y + a_3x^2 + a_4xy = P(t, x, y) \\ y' = b_1x + b_2y + a_3xy + a_4y^2 = Q(t, x, y) \end{cases} \tag{7}$$

and discuss when the function (4) is the reflecting function of the system (7).

Theorem 1 Suppose that $\alpha_i,\ \beta_i,\ \gamma_i\ (i = 1, 2)$ are the solutions of the following equations

$$\beta_1' + (a_1 + \bar{a}_1)\beta_1 + b_1\beta_2 + \bar{a}_2\gamma_1 = 0,\ \beta_1(0) = 1; \tag{8}$$

$$\beta_2' + a_2\beta_1 + (\bar{a}_1 + b_2)\beta_2 + \bar{a}_2\gamma_2 = 0,\ \beta_2(0) = 0; \tag{9}$$

$$\gamma_1' + (a_1 + \bar{b}_2)\gamma_1 + b_1\gamma_2 + \bar{b}_1\beta_1 = 0, \gamma_1(0) = 0; \tag{10}$$

$$\gamma_2' + a_2\gamma_1 + (b_2 + \bar{b}_2)\gamma_2 + \bar{b}_1\beta_2 = 0,\ \gamma_2(0) = 1; \tag{11}$$

$$\alpha_1' = a_3 - a_1\alpha_1 - b_1\alpha_2 + \bar{a}_3\beta_1 + \bar{a}_4\gamma_1,\ \alpha_1(0) = 0; \tag{12}$$

$$\alpha_2' = a_4 - a_2\alpha_1 - b_2\alpha_2 + \bar{a}_3\beta_2 + \bar{a}_4\gamma_2,\ \alpha_2(0) = 0. \tag{13}$$

Then the function (4) is the reflecting function of system (7).

Proof It is easy to see that under the above hypotheses, the function (4) is a solution of the equation (2) with $X(t,x,y) = (P(t,x,y), Q(t,x,y))^T$. Thus, the function (4) is the reflecting function of (7).

Corollary 1 Suppose that the system (7) is a 2ω-periodic system with respect to t and the conditions of Theorem 1 are satisfied. Then the Poincaré mapping of (7) can be expressed as $T(x,y) = F(-\omega,x,y)$ and the solution $(x,y) = (\varphi(t;-\omega,x_0,y_0), \psi(t;-\omega,x_0,y_0))$ defined on $[-\omega,\omega]$ is 2ω-periodic if and only if (x_0,y_0) is a solution of the equations

$$\begin{cases} (\beta_1(-\omega)-1)x + \beta_2(-\omega)y - \alpha_1(-\omega)x^2 - \alpha_2(-\omega)xy = 0 \\ \gamma_1(-\omega)x + (\gamma_2(-\omega)-1)y - \alpha_1(-\omega)xy - \alpha_2(-\omega)y^2 = 0 \end{cases}. \tag{14}$$

Moreover, the system (7) has 1-4 or infinitely 2ω-periodic solutions.

Corollary 2 Assume that

$$a_1 + \bar{a}_1 + b_2 + \bar{b}_2 = 0;$$

$$a_2 e^{-\delta} + \bar{a}_2 e^{\delta} = 0,\ b_1 e^{\delta} + \bar{b}_1 e^{-\delta} = 0,\ \delta = -\int_0^t (b_2 + \bar{b}_2)\mathrm{d}t;$$

$$\hat{\alpha}_1' + \frac{1}{2}(a_1 - \bar{a}_1)\hat{\alpha}_1 + b_1\hat{\alpha}_2 e^{\delta} = a_3 e^{\frac{\delta}{2}} + \bar{a}_3 e^{-\frac{\delta}{2}},\ \hat{\alpha}_1(0) = 0;$$

$$\hat{\alpha}_2' + \frac{1}{2}(b_2 - \bar{b}_2)\hat{\alpha}_2 + a_2\hat{\alpha}_1 e^{-\delta} = a_4 e^{-\frac{\delta}{2}} + \bar{a}_4 e^{\frac{\delta}{2}},\ \hat{\alpha}_2(0) = 0.$$

Then

$$F = \Big(\frac{e^{-\delta}x}{1 + \hat{\alpha}_1 e^{-\frac{\delta}{2}}x + \hat{\alpha}_2 e^{\frac{\delta}{2}}y}, \frac{e^{\delta}y}{1 + \hat{\alpha}_1 e^{-\frac{\delta}{2}}x + \hat{\alpha}_2 e^{\frac{\delta}{2}}y}\Big)^T \tag{15}$$

is the reflecting function of (7).

Proof As $\delta = -\int_0^t (b_2 + \bar{b}_2)\mathrm{d}t$, we have $\delta' = -(b_2 + \bar{b}_2) = a_1 + \bar{a}_1$. Let $\beta_1 = e^{-\delta}$, $\beta_2 = 0$, $\gamma_1 = 0$, $\gamma_2 = e^{\delta}$, $\alpha_1 = \hat{\alpha}_1 e^{-\frac{\delta}{2}}$, $\alpha_2 = \hat{\alpha}_2 e^{\frac{\delta}{2}}$. By the assumptions, we can check that the conditions of Theorem 1 are satisfied. Thus, the present corollary is true.

In Corollary 1, taking $b_1 = 0$ (or $a_2 = 0$ or $a_2 = b_1 = 0$), we get the following result.

Corollary 3 Suppose that

$$a_1 + \bar{a}_1 + b_2 + \bar{b}_2 = 0,\ a_2 e^{-\delta} + \bar{a}_2 e^{\delta} = 0,\ \delta = -\int_0^t (b_2 + \bar{b}_2)\mathrm{d}t.$$

Then the system

$$\begin{cases} x' = a_1 x + a_2 y + a_3 x^2 + a_4 xy \\ y' = b_2 y + a_3 xy + a_4 y^2 \end{cases},$$

has the reflecting function (15), in which

$$\hat{\alpha}_1 = e^{\frac{1}{2}\int_0^t (\bar{a}_1 - a_1)\mathrm{d}t} \int_0^t (a_3 e^{\frac{\delta}{2}} + \bar{a}_3 e^{-\frac{\delta}{2}}) e^{-\frac{1}{2}\int_0^t (\bar{a}_1 - a_1)\mathrm{d}t}\mathrm{d}t,$$

$$\hat{\alpha}_2 = e^{\frac{1}{2}\int_0^t (\bar{b}_2 - b_2)\mathrm{d}t} \int_0^t (a_4 e^{-\frac{\delta}{2}} + \bar{a}_4 e^{\frac{\delta}{2}} - a_2\hat{\alpha}_1 e^{-\delta}) e^{-\frac{1}{2}\int_0^t (\bar{b}_2 - b_2)\mathrm{d}t}\mathrm{d}t.$$

Corollary 4 Suppose that

$$a_1+\bar{a}_1+b_2+\bar{b}_2=0;\ b_1e^{\delta}+\bar{b}_1e^{-\delta}=0,\ \delta=-\int_0^t(b_2+\bar{b}_2)\mathrm{d}t.$$

Then the system

$$\begin{cases} x'=a_1x+a_3x^2+a_4xy, \\ y'=b_1x+b_2y+a_3xy+a_4y^2 \end{cases}$$

has the reflecting function (15), in which

$$\hat{\alpha}_2=e^{\frac{1}{2}\int_0^t(\bar{b}_2-b_2)\mathrm{d}t}\int_0^t(a_4e^{-\frac{\delta}{2}}+\bar{a}_4e^{\frac{\delta}{2}})e^{-\frac{1}{2}\int_0^t(\bar{b}_2-b_2)\mathrm{d}t}\mathrm{d}t,$$

$$\hat{\alpha}_1=e^{\frac{1}{2}\int_0^t(\bar{a}_1-a_1)\mathrm{d}t}\int_0^t(a_3e^{\frac{\delta}{2}}+\bar{a}_3e^{-\frac{\delta}{2}}-b_1\hat{\alpha}_2e^{\delta})e^{-\frac{1}{2}\int_0^t(\bar{a}_1-a_1)\mathrm{d}t}\mathrm{d}t.$$

Corollary 5 Suppose that

$$a_1+\bar{a}_1+b_2+\bar{b}_2=0.$$

Then the Kolmogrov system

$$\begin{cases} x'=a_1x+a_3x^2+a_4xy \\ y'=b_2y+a_3xy+a_4y^2 \end{cases} \tag{16}$$

has the reflecting function

$$F=\Big(\frac{e^{-\int_0^t(a_1+\bar{a}_1)\mathrm{d}t}x}{1+\alpha_1x+\alpha_2y},\frac{e^{-\int_0^t(b_2+\bar{b}_2)\mathrm{d}t}y}{1+\alpha_1x+\alpha_2y}\Big)^T,$$

in which

$$\alpha_1=e^{-\int_0^t a_1\mathrm{d}t}\int_0^t(a_3e^{\int_0^t a_1\mathrm{d}t}+\bar{a}_3e^{\int_0^{-t} a_1\mathrm{d}t})\mathrm{d}t,$$

$$\alpha_2=e^{-\int_0^t b_2\mathrm{d}t}\int_0^t(a_4e^{\int_0^t b_2\mathrm{d}t}+\bar{a}_4e^{\int_0^{-t} b_2\mathrm{d}t})\mathrm{d}t.$$

Example 1 It is not difficult to check that the function

$$F=\Big(\frac{e^{-2\sin t}x}{1+2te^{\cos t-\sin t}x+2te^{\sin t+\cos t}y},\frac{e^{2\sin t}y}{1+2te^{\cos t-\sin t}x+2te^{\sin t+\cos t}y}\Big)^T$$

is the reflecting function of the system

$$\begin{cases} x'=x(\sin t+\cos t+e^{\cos t-\sin t}x+e^{\sin t+\cos t}y) \\ y'=y(-\cos t+\sin t+e^{\cos t-\sin t}x+e^{\sin t+\cos t}y) \end{cases}. \tag{17}$$

As this system is a 2π-periodic system, its Poincaré mapping can be express by $T(x,y)=F(-\pi,x,y)=\Big(\frac{x}{1-2\pi e^{-1}(x+y)},\frac{y}{1-2\pi e^{-1}(x+y)}\Big)^T$. Thus, all the solutions $(x,y)=(\varphi(t;-\pi,x_0,-x_0),\psi(t;-\pi,x_0,-x_0))$ of (17) defined on $[-\pi,\pi]$ are 2π-periodic.

Example 2 The differential system

$$\begin{cases} x'=x(2\sin t+\cos t)+y\cos t+(2\sin^2 t+\cos t)x(x+y) \\ y'=-x\cos t+y(2\sin t-\cos t)+(2\sin^2 t+\cos t)y(x+y) \end{cases},$$

has the reflecting function

$$F = \Big(\frac{(1-2\sin t)x - 2y\sin t}{1+2\sin t(x+y)}, \frac{2x\sin t + (1+2\sin t)y}{1+2\sin t(x+y)}\Big)^T,$$

and all its solutions defined on $[-\pi, \pi]$ are 2π-periodic.

Remark 1 By the equations (8-11), we know that the matrix

$$M(t) = \begin{pmatrix} \beta_1 & \beta_2 \\ \gamma_1 & \gamma_2 \end{pmatrix} \tag{18}$$

is the reflecting matrix of the linear system

$$z' = A(t)z, \tag{19}$$

where $A(t) = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}$. Let $\alpha = (\alpha_1, \alpha_2)^T$, $b = (a_3, a_4)^T$. Then Theorem 1 can be rewritten as

Theorem 2 If

$$\Phi(t, x) = M^T(t)z + \alpha$$

is the reflecting function of the linear system

$$z' = \bar{A}^T z - \bar{b},$$

then the function (4) is the reflecting function of the system (7).

Remark 2 By the reference [14], we know that the matrix (18) can be expressed as

$$M(t) = G(t)e^{\sigma},\ \sigma = -\int_0^t \mathrm{tr}A_e(t)\mathrm{d}t, \tag{20}$$

where

$$G(t) = \begin{pmatrix} g_1 & g_2 \\ g_3 & \bar{g}_1 \end{pmatrix},\ g_1\bar{g}_1 - g_2 g_3 = 1,\ g_2 + \bar{g}_2 = 0,\ g_3 + \bar{g}_3 = 0,\ g_1(0) = 1,$$

$$A_e = \frac{A(t) + A(-t)}{2},\ A = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}.$$

Theorem 3 Suppose that $G(t)$ is the reflecting matrix of the linear system

$$u' = Bu,\ B = \begin{pmatrix} a^* & a_2 \\ b_1 & -a^* \end{pmatrix},\ a^* = \frac{a_1 - b_2}{2} \tag{21}$$

and

$$\alpha' + A^T\alpha = b + M^T\bar{b},$$

where $M = Ge^{\sigma}$. Then the function (4) is the reflecting function of (7).

By the reference [14], we see that the matrix

$$G = \begin{pmatrix} 1+S^2 & S \\ S(2+S^2) & 1+S^2 \end{pmatrix}$$

is the reflecting matrix of the linear system

$$u' = \begin{pmatrix} \frac{1}{2}S^3C - S^2 & -\frac{1}{2}C^3 - 2S \\ \frac{1}{2}C(S^4+S^2+2) - 2S & -\frac{1}{2}S^3C + S^2 \end{pmatrix} u,$$

where $S = \sin t,\ C = \cos t$.

Example 3 The differential system

$$\begin{cases} x' = \Big(\dfrac{1}{2}S^3C - S^2\Big)x - \Big(\dfrac{1}{2}C^3 + 2S\Big)y + a_3x^2 + a_4xy \\ y' = \Big(\dfrac{1}{2}C(S^4 + S^2 + 2) - 2S\Big)x + \Big(-\dfrac{1}{2}S^3C + S^2\Big)y + a_3xy + a_4y^2 \end{cases}, \tag{22}$$

where

$$a_3 = -S^4 - \frac{SC}{2(1+S^2)}(14 + 13S^2 + 5S^4 + 2S^6),$$

$$a_4 = -\frac{SC}{1+S^2}(2 + S^2 + S^4),\ (S := \sin t,\ C := \cos t)$$

has the reflecting function

$$F = \Big(\frac{(1+S^2)x + Sy}{1 + S^4x + S^3y}, \frac{S(2+S^2)x + (1+S^2)y}{1 + S^4x + S^3y}\Big)^T.$$

As $F(-\pi, x, y) \equiv (x, y)^T$, all the solutions of the present quadratic system defined on $[-\pi, \pi]$ are 2π-periodic.

Similar to [14], we can get many sufficient conditions under which the matrix (19) is the reflecting matrix of the linear system (20).

In general, we can extend the conclusions of Theorem 1.

Theorem 4 If the matrix $M = (m_{ij}(t))_{n\times n}$ is the reflecting matrix of the linear system $z' = A(t)z$ and

$$\alpha' + A(t)\alpha = b + M^T\bar{b},$$

then the function

$$F = \Big(\frac{\sum_{i=1}^n m_{1i}x_i}{1 + \sum_{i=1}^n \alpha_i x_i}, \frac{\sum_{i=1}^n m_{2i}x_i}{1 + \sum_{i=1}^n \alpha_i x_i}, ..., \frac{\sum_{i=1}^n m_{ni}x_i}{1 + \sum_{i=1}^n \alpha_i x_i}\Big)^T$$

is the reflecting function of the system

$$x' = A(t)x + x\sum_{i=1}^n b_i x_i,$$

where $A(t) = (a_{ij}(t))_{n\times n},\ b = (b_i(t))_{n\times 1},\ \alpha = (\alpha_i(t))_{n\times 1},\ x = (x_i)_{n\times 1}$.

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非线性与线性微分系统的等价关系研究

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摘　要：建立了一类时变非线性微分系统与线性微分系统之间的等价关系，给出了一类二次微分系统具有有理分式形式的反射函数的充分条件。并应用所得结论研究了二次微分系统及时变 Kolmogrov 系统的周期解及其定性性态.

关键词：反射函数；周期解；时变系统

(责任编辑：冯珍珍，包震宇)