Analysis of a stage-structured mosquito population model

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Abstract: In order to study the impact of limited resource and maturation time on mosquito population dynamics, we formulate a simple stage-structured mosquito population model with saturated recruitment rate, where mosquito population is divided into two class. The basic investigation of the model, such as the existence of equilibria and their stability, has been finished. The global stability of the unique endemic equilibrium has been proved. A numerical simulation has been presented too. Our findings finally show that reducing the resource and maturation time, are effective methods to control the number of mosquitoes.

Key words: mosquito; resource; maturation time; stage-structured model; stability

一类阶段结构蚊子种群的分析

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摘　要：为了研究有限的资源以及蚊子幼虫成熟时间对蚊子种群的影响，我们建立了一个带有饱和增长率的蚊子种群模型并对该模型进行了基本的数学分析，包括平衡点的存在性及其稳定性。另外，我们还进行了一些关键参数的敏感性分析. 研究结果表明减少资源和幼蚊的成熟时间，是控制蚊子数量的有效方法.

关键词：蚊子；资源；成熟时间；阶段结构模型；稳定性

1   Introduction

Mosquito is the vector of many diseases, like malaria, dengue, West Nile virus, etc. As we know, mosquito-borne diseases kill more people than any other diseases caused by single factor. In order to control these diseases, it is essential to understand mosquito population dynamics, to consider how resource and temperature can affect mosquito population, and affect mosquito-borne disease transmission.

Mosquito life includes four stages: egg, larva, pupa, and adult. Each of these stages can be easily recognized by their special appearance. The duration of the whole cycle, from egg laying to an adult mosquito eclosion, varies between 7 and 20 days, depending on the ambient temperature of the swamp and the mosquito species involved[1]. Only the adult female mosquitoes bite human beings and animals in order to take blood meals, and the male mosquitoes feed only on plant juices. Female mosquitoes feed on man, domestic animals, such as cattle, horses, goats, etc; all species of birds including chickens and ducks; all types of wild animals including deer, rabbits; and they also feed on snakes, lizards, frogs, and toads. Female mosquitoes will not lay viable eggs without blood meals. Therefore, the abundance of mosquito in a region is closely related to the blood meal resources available in the region[2].

Since only the adult female mosquitoes are responsible for transmitting diseases, therefore in general, models focus only on describing the dynamics of adult female mosquitoes. There have been extensive dynamical modeling studies of the mosquito population and mosquito-borne diseases (see [3–12] etc.)

We note that few models mentioned above includes the immature mosquito stage which is more sensitive to climate change and resource. It is our aim to formulate a mosquito population model to include the immature stage to investigate the impact of limited resource and maturation time on the transmission dynamics of mosquito-borne diseases.

The rest of the paper is organized as follows. In Section 2, the derivation of a new model is given. We analyze the dynamics of this model in Section 3, including the existence and stability of equilibria. Finally, we present some numerical simulations and comments on our findings in Section 4.

2   Model

We group the three aquatic stages of mosquitoes into one class and divide the mosquito population into only two classes. one class consists of the first three stages, denoted by J, and the other one of which consists of all adults, denoted by N. In order to depict the impact of limited resource on mosquito reproduction, we let the birth function, that is, the oviposition function of adults be $\dfrac{b_v N}{1 + N}$. We let α be the maturation rate of larvae. The death rate of larvae is a linear function $(d_0 + d_1 J)$, which d_0 and d_1 are the density independent and dependent coefficients, respectively, and the death rate of adults is a constant μ_v. The stage-structured population dynamics of mosquitoes is given by:

$$\begin{cases} \dfrac{\mathrm{d}J}{\mathrm{d}t} = \dfrac{b_v N}{1 + N} - \alpha J - d_0 J - d_1 J^2 \ , \\[2mm] \dfrac{\mathrm{d}N}{\mathrm{d}t} = \alpha J - \mu_v N \ . \end{cases} \tag{1}$$

3   Dynamical analysis

Obviously, $(0, 0)$ is an equilibrium of (1). The local stability of the trivial equilibrium can be determined by the eigenvalues of the Jacobian matrix at $(0, 0)$ which has the form of

$$J_0 = \begin{pmatrix} -(\alpha + d_0) & b_v \\ \alpha & -\mu_v \end{pmatrix} \ .$$

Define the net reproductive number for system (1) as

$$R_0 = \frac{b_v \alpha}{\mu_v(\alpha + d_0)}.$$

It is easy to show that $\mathrm{tr} J_0 < 0$ and $\det J_0 > 0 \iff R_0 < 1$. Hence, we have the following result:

Theorem 1   The trivial equilibrium (0,0) is locally asymptotically stable if $R_0 < 1$. On the other hand, (0,0) is unstable if $R_0 > 1$.

If there exists a positive equilibrium of system (1), it satisfies

$$\frac{b_v N}{1 + N} = \alpha J + (d_0 + d_1 J)J, \tag{2a}$$

$$\alpha J = \mu_v N. \tag{2b}$$

Substituting $N = \frac{\alpha J}{\mu_v}$ into the first equation (2a) yields

$$\alpha d_1 J^2 + (\mu_v d_1 + \alpha d_0 + \alpha^2)J + [\mu_v(\alpha + d_0) - b_v \alpha] = 0 . \tag{3}$$

Hence, there exists a unique positive solution, denoted by (J^*, N^*), if and only if $\mu_v(\alpha + d_0) - b_v \alpha < 0$ which implies $R_0 = \dfrac{b_v \alpha}{\mu_v(\alpha + d_0)} > 1$.

We next work on the stability of the positive equilibria.

The local stability of the positive equilibrium (J^*, N^*) can be determined by the eigenvalues of the Jacobian matrix at the positive equilibrium which has the form of

$$J^+ = \begin{pmatrix} -(\alpha + d_0 + 2d_1 J^*) & \dfrac{b_v}{(1 + N^*)^2} \\ \alpha & -\mu_v \end{pmatrix} .$$

It is easy to show that $\mathrm{tr} J^+ < 0$.

And, the equations (2a) and (2b) lead to

$$\alpha + d_0 = -d_1 J^* + \frac{b_v \alpha}{\mu_v + \alpha J^*} .$$

Therefore,

$$\begin{aligned}
\det J^+ &= \mu_v(\alpha + d_0) + 2\mu_v d_1 J^* - \frac{b_v \alpha}{(1 + N^*)^2} \\
&= \mu_v(\alpha + d_0) + 2\mu_v d_1 J^* - \frac{b_v \alpha \mu_v{}^2}{(\mu_v + \mu_v N^*)^2} \\
&= \mu_v \left((\alpha + d_0) + 2d_1 J^* - \frac{b_v \alpha \mu_v}{(\mu_v + \alpha J^*)^2} \right) \\
&= \mu_v \left[d_1 J^* + \frac{b_v \alpha(\mu_v + \alpha J^*) - b_v \alpha \mu_v}{(\alpha J^* + \mu_v)^2} \right] \\
&= \mu_v \left(d_1 J^* + \frac{\alpha^2 b_v J^*}{(\alpha J^* + \mu_v)^2} \right) > 0.
\end{aligned} \tag{4}$$

Hence the positive equilibrium is a locally asymptotically stable node.

For simplicity, we denote the two expression on the right hand side of system (1) by $f_1(J_v, N_v)$ and $f_2(J_v, N_v)$.

Then, it is easy to check that

$$\partial_J f_1(J, N) + \partial_N f_2(J, N) = -(\alpha + d_0 + 2\mu_v d_1 J + \mu_v) < 0.$$

Thus, by the Bendixson-Dulac Principle, system (1) has no closed orbits in the positive quadrant of JN-plane. So, the positive equilibrium is globally asymptotically stable. In summary, we have the following results.

Theorem 2   If $R_0 < 1$, the trivial equilibrium (0,0) of system (1) is a locally asymptotically stable node, and there exists no positive equilibrium. If $R_0 > 1$, the trivial equilibrium (0,0) of system (1) is unstable, and there exists a unique positive equilibrium, which is globally asymptotically stable.
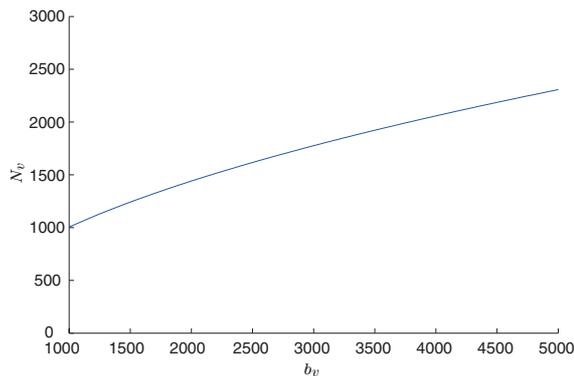
4   Discussion

Table 1 Description of parameters of the models

Interpretation	Parameter	Range	Reference
The maximum value of the recruitment rate of viable mosquito eggs	b_v	[1000, 5000]	[13, 14]
The duration of the whole cycle, from egg laying to an adult mosquito eclosion	$1/\alpha$	[7, 20] days	[16]
Natural death rate of immature mosquitoes	d_0	0.2	[15]
Natural death rate of adult mosquitoes	μ_v	0.03	[15]
Density-dependent mortality of immature mosquitoes	d_1	0.01	[15]
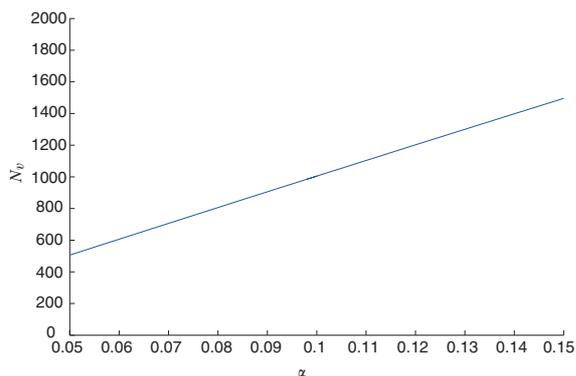
(a) The abscissa is α and the ordinate is N_v. ($b_v = 1000$).   (b) The abscissa is b_v and the ordinate is N_v. ($\alpha = 0.1$).

Figure 1 The value of adult mosquitoes at the positive equilibrium. The parameter values are depicted in Table 1 except for α and b_v.

In this paper, in order to study the impact of limited resource and maturation time on the population dynamics of mosquitoes, we formulated a stage-structured mosquito model. The formula of the reproductive number for mosquitoes has been derived. We also explored the existence and stability of equilibria. Especially, the global stability of the unique positive equilibrium has been proved.

In order to study the impact of maturation time and limited resource, we draw the curve of the number of adult mosquitoes at the positive equilibrium using some parameter values listed in Table 1 with respect to α and b_v, respectively. Simulation result Fig.1(a) manifests that the number of mosquitoes is a monotonic increasing function of α which implies that with the increasing of temperature, the maturation time $1/\alpha$ becomes smaller, there will be more mosquitoes in a given region. Another result Fig.1(b) tells us that the abundance of the resource related to mosquito reproduction affects the number of mosquitoes positively. Some human behaviors, like pouring water, spraying insecticide, which can reduce the abundance of resource, are effective methods to control the number of mosquitoes and in turn to control mosquito-borne diseases.

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