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The existence of n-order algebraic curve solutions of planar quadratic polynomial systems

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Abstract: In hi ho a icle, e ho ha fo any giv en po i iv e in ege $n \geqslant 2$, he e i a plana ad a ic diffe en ial y em hav ing n-deg ee and $2n-$deg ee cla ical algeb aic c v e ol ion .

Key words: ad a ic diffe en ial y em; algeb aic c v e ol ion; pe iodic ol ion; in eg able y em

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平面二次多项式系统中 n 阶代数曲线解的存在性

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摘 要：用具体例子证明对于任给的正整数 $n \geqslant 2$, 平面二次多项式系统中，存在 n 阶和 $2n$ 阶代数曲线解.

关键词：二次系统；代数曲线解；周期解；可积系统

1 Introduction

We a e in e e ed in he dy of plana poly nomial y em , beca e hey occ v ey of en in applica ion . Indeed, ch e a ion appea in modelling chemical eac ion , pop la ion dy namic , av elling av e y em of nonlinea ev ol ion e a ion in ma hema ical phy ic and in many o he a ea of applied ma hema ic and mechanic . F om he ma hema ical poin of v ie , ad a ic y em a e pe hap he mo imple nonlinea diffe en ial y em . De pi e hei implici y , he e a e impo an open e ion a o nd hem. I i a pa ic la ca e of he famo Hilbe ′ 16 h p oblem[1].

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Mo e ecen ly, Ga cia and Llib e[2] gave a lo of example o ho ha cla ical plana algeb aic c ve a e eali able by ad a ic polynomial diffe en ial y em . We ay ha a given algeb aic c ve i eali ed by a ad a ic y em hen hi algeb aic c ve i an inva ian c ve ol ion of a y em

$$\frac{\mathrm{d}x}{\mathrm{d}t}=P_2(x,y)=\Sigma_{i+j=0}^{2}a_{ij}x^iy^j,\quad \frac{\mathrm{d}y}{\mathrm{d}t}=Q_2(x,y)=\Sigma_{i+j=0}^{2}b_{ij}x^iy^j, \tag{1}$$

he e a_{ij} and b_{ij} a e con an pa ame e .

I i ell kno n ha any c bic algeb aic c ve i al ay eali ed by ome ad a ic Hamil onian y em[3−4]. Q ad a ic y em eali ing cla ical ad a ic and highe deg ee algeb aic c ve have been fo nd.

Fo a given po i ive in ege n, i he e an n-deg ee algeb aic c ve ch ha i i eali able by ad a ic poly-nomial diffe en ial y em ? In hi pape , e ho he follo ing concl ion.

Theorem 1 Fo any given po i ive in ege $n\geqslant 2$, he e i a plana ad a ic diffe en ial y em having n-deg ee and $2n-$deg ee cla ical algeb aic c ve ol ion .

The p oof of hi heo em i given in ec ion 2.

2 An integrable quadratic system having n-degree classical algebraic curve solution

Con ide he ad a ic diffe en ial y em

$$\frac{\mathrm{d}x}{\mathrm{d}t}=-xy,\quad \frac{\mathrm{d}y}{\mathrm{d}t}=ax^2+by^2-\frac{1}{4a}. \tag{2}$$

Thi i an in eg able y em i h he fi in eg al

$$H(x,y)=x^{2b}\left(y^2+\frac{a}{b+1}x^2-\frac{1}{4ab}\right)=h,\quad \text{fo}\quad b\neq -1; \tag{3}$$

$$H(x,y)=\frac{y^2}{x^2}+\frac{1}{4ax^2}+2a\ln(x),\quad \text{fo}\quad b=-1. \tag{4}$$

We only con ide he ca e $a>0$. O he i e, e apply he an fo ma ion $x\to -x, y\to -y$.

Clea ly, y em (2) al ay ha o ing la poin $E_{\mp}\left(\mp\frac{1}{2a},0\right)$. When $ab>0$, he e exi o ing la poin $S_{\mp}\left(0,\mp\frac{1}{2\sqrt{ab}}\right)$ in he aigh line $x=0$.

Le $M(x_j,y)$ be he coefficien ma ix of he linea i ed y em fo e a ion (2) a a ing la poin . We have

$$J\left(\mp\frac{1}{2a},0\right)=\det M\left(\mp\frac{1}{2a},0\right)=\frac{1}{2a}>0,\ J\left(0,\mp\frac{1}{2\sqrt{ab}}\right)=\det M\left(0,\mp\frac{1}{2\sqrt{ab}}\right)=-\frac{1}{2a}<0.$$

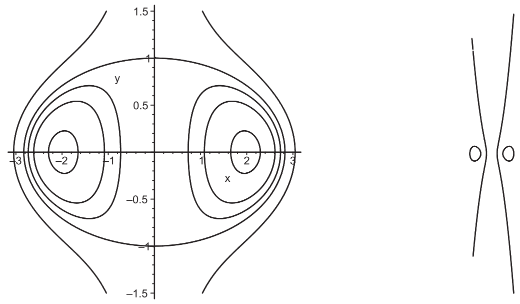
When $b\neq -1$, e i e ha

$$h_1=H\left(-\frac{1}{2a},0\right)=-\frac{1}{4a}\left(-\frac{1}{2a}\right)^{2b}\left(\frac{1}{b}-\frac{1}{b+1}\right),$$

$$h_2=H\left(\frac{1}{2a},0\right)=-\frac{1}{4a}\left(\frac{1}{2a}\right)^{2b}\left(\frac{1}{b}-\frac{1}{b+1}\right),\ h_s=H\left(0,\frac{1}{2\sqrt{ab}}\right)=0.$$

By ing he above info ma ion, e have he follo ing ali a ive analy i .

Lemma 1 The plana ad a ic y em (2) ha o y mme ic cen e i h e pec o he $y-$coo dina e axi , fo hich he e exi exac ly h ee opological pha e po ai ho n in Fig . 1(a), 1(b) and 1(c).



(a) $b > 0$