

# The dynamics of poverty and crime

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**Abstract:** Poverty and crime are two maladies that plague metropolitan areas. The economic theory of crime<sup>[1]</sup> demonstrates a direct correlation between poverty and crime. The model considered in this study seeks to examine the dynamics of the poverty-crime system through stability analysis of a system of ordinary differential equations in order to identify cost-effective strategies to combat crime in metropolises.

**Key words:** Mathematical model; Poverty and crime; Crime control strategies

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## 1 Introduction

There is a direct correlation between the poverty and criminality<sup>[2-3]</sup>. Becker's economic theory of crime<sup>[1]</sup> assumes that people resort to crime only if the costs of committing the crime are lower than the benefits gained. Those living in poverty, therefore, have a much greater chance of committing property crime<sup>[3-4]</sup> than the general population, as they stand to gain more with each crime. Property crime includes the offenses of burglary, larceny-theft, motor vehicle theft, and arson<sup>[5]</sup>. In his 1968 paper<sup>[1]</sup>, Becker used statistical and economic analysis to determine the optimal control of crime. Here, we use a system of ordinary differential equations (ODEs) to try and get a more realistic, dynamical solution to that same question.

Property crime is a major problem in metropolises. In the Bronx borough of New York City alone there are nearly 45,000 cases of property crime reported in 2013<sup>[6]</sup> in a population of over 1.4 million<sup>[7]</sup>. Each criminal costs society about \$5,700 per year due to lost productivity<sup>[8]</sup>, and a total of \$24 billion in goods is lost in the US each year to property crime<sup>[9]</sup>. The victims of crime suffer an aggregate burden of \$472 billion per year, including mental and physical suffering<sup>[10]</sup>.

Crime is clearly an important problem that must be confronted. Ehrlich<sup>[11]</sup> suggests that the successfulness of rehabilitation and incapacitation programs do have an effect on the aggregate crime level. However, it costs \$20,000 – \$30,000 to detain a person in a federal prison each year<sup>[12]</sup>. It also costs about \$15,000 – \$20,000 annually for a prison cell<sup>[9]</sup>. We see that from an economic standpoint, detaining every prisoner is actually a greater burden on society than crime itself is. Therefore, the issue becomes one of balance. Ultimately,

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the goal is to reduce crime to such a level that the total cost of controlling crime and the cost of the crime that remains is less than the total cost of crime under the status quo.

One of the main objectives of this modeling study is to find a cost-effective strategy to lower criminality, thus lowering the cost of crime to society. Previous works have addressed this problem<sup>[1,13]</sup> using statistical and economic approaches, while we do so by taking a mathematical approach. The use of ODEs allows us to examine the interactions between poverty, crime and intervention, which provides a dynamic, rather than static, view of how criminality is affected by varying intervention parameters.

It is known that the problem of crime is alleviated by either decreasing poverty<sup>[14]</sup> or by increasing the severity of the ensuing punishment<sup>[15]</sup>. Our model considers both intervention measures concomitantly. Logically, crime will decrease if one or the other intervention parameters is increased. However, because we seek a pragmatic solution in a world where resources are limited and cost is always a consideration, we cannot reduce crime by simply relieving all poverty or by incarcerating all criminals. Instead, we seek a cost-effective strategy to combat crime. Our model shows that the optimal solution is actually a combination of the two control measures and pinpoints where that optimal solution is.

## 2 Model

Naturally, not all crime can be stopped; that would not be economically desirable. This model seeks to optimize interventions so that crime is reasonably controlled while the cost is minimal or crime level is minimal under a given constraint on cost. The two intervention measures considered in the model are represented by  $\gamma$ , the rate of converting those in poverty to recovered, and  $\rho$ , the rate of incarceration, which can vary due to the change in the effort of enforcement. In the model, the population is divided into five sub-classes: the non-impooverished class  $N$ , the poverty class  $P$ , the criminal class  $C$ , the jailed class  $J$ , and recovered class (from jail or from impooverished class)  $R$ . Let these variables represent the fractions of the sub-populations at time  $t$ . Then

$$N(t) + P(t) + C(t) + J(t) + R(t) = 1.$$

Let  $\sigma$  denote the rate of the flow from the non-impooverished class to the impooverished class. It is assumed that  $\sigma$  is omnipresent and dependent upon the unemployment rate because of the nature of unemployment and because of the dependency of poverty on unemployment. Let  $\gamma$  denote the conversion rate from the  $P$  class to the  $R$  class due to government interventions;  $\rho$  denote the rate at which criminals are captured and  $\varepsilon$  denote the fraction of these criminals who ended up in jail;  $\delta$  denote the rate at which individuals get out of jails;  $\mu$  denote the rate at which individuals enter and exit the population. All rates are per capita and are positive constants. The interaction between poverty and crime is governed by the following system of differential equations:

$$\begin{aligned} N' &= \mu T - (\sigma + \mu)N, \\ P' &= \sigma N - \beta PC - (\gamma + \mu)P, \\ C' &= \beta PC + \phi \beta RC - (\varepsilon \rho + \mu)C, \\ J' &= \varepsilon \rho C - (\delta + \mu)J, \\ R' &= \gamma P + \delta J - \phi \beta RC - \mu R. \end{aligned} \quad (1)$$

Here “'” denotes  $d/dt$ . The model assumes that there is a certain probability that a person in the  $P$  class will resort to crime after coming into contact with a criminal. The term  $\beta PC$  is the conversion of impooverished individuals to criminals due to contact over a certain period of time, and  $\beta$  represents the “transmission” rate. A recovered individual may also become criminal again but at a reduced rate  $\phi \beta RC$  where  $0 \leq \phi \leq 1$  is the reduction fraction that accounts for recidivism. According to the report from Bureau of Justice Statistics, about two-thirds (67.8%) of released prisoners were arrested for a new crime within 3 years, and three-quarters

(76.6%) were arrested within 5 years<sup>[16]</sup>. The assumption is that those who have already gone to jail and entered the  $R$  class may revert back to criminality at some reduced rate  $\phi\beta$  due to contact with criminals. The rate is reduced because these people have a greater cost to commit their next crime according to Becker's theory<sup>[1]</sup>. A list of all variables and parameters are provided in Table 1.

Table 1 Definition of symbols frequently used in the model analysis and simulations. All rates are per-capita

Symbol	Definition
$t$	Time
$N$	Fraction of non-impoveryshed individuals
$P$	Fraction of individuals in the poverty class
$C$	Fraction of individuals in the criminal class
$J$	Fraction of individuals in the jailed class
$R$	Fraction of individuals in the recovered class (from $P$ or $J$ )
$\beta$	Rate at which a $P$ individual becomes $C$ by contacting them
$\phi$	Reduction factor for $R$ to become crimial influenced by $C$
$\sigma$	Rate at which an $N$ individual enters the $P$ class
$\gamma$	Conversion rate from the $P$ class to the $R$ class
$\rho$	Rate at which criminals are captured
$\varepsilon$	Proportion of captured criminals that are jailed
$\delta$	Rate at which a $J$ individual gets out jails
$\mu$	Rate of recruitment and exit of the population
$C^*(\gamma, \rho)$	Fraction of criminal population at the positive equilibrium
$C_0^*$	$= C^*(\gamma_0, \rho_0)$ crime activity under the baseline policy
$\mathcal{C}(\gamma, \rho)$	Cost associated with intervention parameters $\gamma$ and $\rho$
$\mathcal{C}_0$	$= \mathcal{C}(\gamma_0, \rho_0)$ cost correspong to the baseline policy ( $\gamma_0, \rho_0$ )

### 3 Mathematical analysis

In this section we derive threshold conditions that determine the population dynamics of the system (1), and these threshold conditions can be helpful for identifying crime control strategies.

Let  $\mathbf{R}$  denote the reproduction number of criminals, which is defined as the number of secondary criminal cases generated by one typical criminal individual during the entire period of criminality when introduced into a non-impoveryshed population. It can be calculated from model (1) that the reproductive number is given by

$$\mathbf{R} = \mathbf{R}_P + \mathbf{R}_R, \quad (2)$$

where

$$\begin{aligned} \mathbf{R}_P &= \left( \frac{\beta}{\varepsilon\rho + \mu} \right) \left( \frac{\sigma}{\sigma + \mu} \right) \left( \frac{\mu}{\gamma + \mu} \right), \\ \mathbf{R}_R &= \left( \frac{\phi\beta}{\varepsilon\rho + \mu} \right) \left( \frac{\sigma}{\sigma + \mu} \right) \left( \frac{\gamma}{\gamma + \mu} \right). \end{aligned} \quad (3)$$

$\mathbf{R}_P$  and  $\mathbf{R}_R$  represent the contributions from the  $P$  and  $R$  classes, respectively. The factors  $\beta/(\varepsilon\rho + \mu)$  and  $\phi\beta/(\varepsilon\rho + \mu)$  give the numbers of new criminals from the  $P$  and  $R$  classes, respectively, produced by one criminal individual during the entire criminal period before being captured. The factor  $\sigma/(\sigma + \mu)$  gives the probability that a non-impoveryshed individual survived and entered the impoveryshed class while  $\mu/(\gamma + \mu)$  and  $\gamma/(\gamma + \mu)$  represent respectively the probabilities that a  $P$  individual remains in the  $P$  class or has moved

into the  $R$  class. As shown below  $\mathbf{R} = 1$  is a threshold value which determines whether the population size of criminals will go to zero or establish at a positive equilibrium as  $t \rightarrow \infty$ . This is shown by studying the existence and stability of equilibria of the system (1).

System (1) always has the crime-free equilibrium (CFE) denoted by  $E_0 = (N_0, P_0, C_0, J_0, R_0)$ :

$$E_0 = \left( \frac{\mu}{\sigma + \mu}, \frac{\mu\sigma}{(\sigma + \mu)(\gamma + \mu)}, \rho, \rho, \frac{\gamma\sigma}{(\sigma + \mu)(\gamma + \mu)} \right). \quad (4)$$

Let  $E^* = (N^*, P^*, C^*, J^*, R^*)$  with  $C^* > 0$  denote the positive crime activity at an equilibrium. Setting the right-hand-side of equations in (1) equal to zero we can express all components of  $E^*$  in terms of  $x = C^*$ :

$$\begin{aligned} N^* &= \frac{\mu}{\sigma + \mu}, \quad P^* = \frac{\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta x)}, \quad J^* = \frac{\varepsilon\rho x}{\delta + \mu}, \\ R^* &= \frac{1}{\mu + \phi\beta x} \left( \frac{\gamma\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta x)} + \frac{\delta\varepsilon\rho x}{\delta + \mu} \right). \end{aligned} \quad (5)$$

Using the fact that  $N + P + C + J + R = 1$  (and noting that  $N^* = \mu/(\sigma + \mu)$ ), we get the equation for  $x$ :

$$\begin{aligned} \frac{\mu}{\sigma + \mu} + \frac{\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta x)} + x + \frac{\varepsilon\rho x}{\delta + \mu} \\ + \frac{1}{\mu + \phi\beta x} \left( \frac{\gamma\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta x)} + \frac{\delta\varepsilon\rho x}{\delta + \mu} \right) = 1, \end{aligned} \quad (6)$$

or equivalently

$$(a_2x^2 + a_1x + a_0)x = 0, \quad (7)$$

where

$$\begin{aligned} a_0 &= (\gamma + \mu)(\varepsilon\rho + \mu)(1 - \mathbf{R}), \\ a_1 &= \phi\beta(\gamma + \mu) \left( 1 + \frac{\varepsilon\rho}{\delta + \mu} \right) + \beta(\varepsilon\rho + \mu) \left( 1 - \frac{\gamma + \mu}{\gamma} \mathbf{R}_R \right), \\ a_2 &= \left( 1 + \frac{\varepsilon\rho}{\delta + \mu} \right) \phi\beta^2. \end{aligned} \quad (8)$$

Equation (7) has one solution  $x = 0$  and two other solutions which we denote by  $x_{\pm}$  given by

$$x_{\pm} = \frac{1}{2a_2} (-a_1 \pm \sqrt{a_1^2 - 4a_0a_2}).$$

Notice that  $a_2 > 0$  for all parameter values. The sign of  $a_0$  depends on the magnitude of  $\mathbf{R}$ . We consider two cases.

**Case 1**  $\mathbf{R} > 1$ . In this case  $a_0 < 0$  and  $a_0a_2 < 0$ . Hence  $x_{-}$  is always negative while  $x_{+}$  is always positive. It follows that equation (7) has a unique positive solution; and thus there is a unique positive equilibrium  $E^*$ .

**Case 2**  $\mathbf{R} < 1$ . In this case  $a_0 > 0$  and  $a_0a_2 > 0$ . If the duration in the poverty class  $1/\gamma$  is smaller than the total duration in the population  $1/\mu$  (which is true in general) and

$$\mathbf{R}_R < 1 - \frac{\mu}{\gamma + \mu} \quad (9)$$

(note that  $\mathbf{R}_R < \mathbf{R} < 1$ ) then  $a_1 > 0$  in which case  $x_{\pm}$  are either negative or complex. Thus there is no positive equilibrium.

**Result 1** When  $\mathbf{R} > 1$  system (1) has a unique crime equilibrium  $E^*$ . When  $\mathbf{R} < 1$  and condition (9) holds there is only the CFE  $E_0$ .

We proceed to show the stability of the equilibria. At the CFE  $E_0$  the Jacobian matrix is

$$J(E_0) = \begin{bmatrix} -(\sigma + \mu) & 0 & 0 & 0 & 0 \\ \sigma & -(\gamma + \mu) & -(\varepsilon \rho + \mu)R_P & 0 & 0 \\ 0 & 0 & (\varepsilon \rho + \mu)(R - 1) & 0 & 0 \\ 0 & 0 & \varepsilon \rho & -(\delta + \mu) & 0 \\ 0 & \gamma & -(\varepsilon \rho + \mu)R_R & \delta & -\mu \end{bmatrix}. \tag{10}$$

$J(E_0)$  has four negative eigenvalues  $\lambda_1 = -(\sigma + \mu)$   $\lambda_2 = -(\gamma + \mu)$   $\lambda_3 = -(\delta + \mu)$   $\lambda_4 = -\mu$  and the fifth eigenvalue is given by

$$\lambda_5 = (\varepsilon \rho + \mu)(R - 1).$$

Hence all eigenvalues of  $J(E_0)$  are negative if  $R < 1$ , and  $J(E_0)$  has one positive eigenvalue if  $R > 1$ . Fig. 1 (a) plots the numerical solutions of system (1) for the case  $R < 1$ . We observe that  $e(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Fig. 1 (b) is for the case  $R > 1$ . It shows that the solution converges to a positive steady state, showing that  $E_0$  is unstable. Thus the following result holds.

**Result 2** The CFE equilibrium  $E_0$  is locally asymptotically stable (l. a. s.) if  $R < 1$  and it is unstable if  $R > 1$ .

The characteristic equation for the crime equilibrium  $E^*$  is a degree 5 polynomial for which it is difficult to obtain analytical results. We explore the stability of  $E^*$  numerically. Fig. 1 (b) is a time plot of numerical solutions of system (1) for the case  $R > 1$ . It shows that the solution converges to  $E^*$  as  $t \rightarrow \infty$ . The parameter values used are:  $\beta = 2.8$   $\phi = 0.1$   $\mu = 0.1$   $\sigma = 0.5$   $\delta = 1/5$   $\varepsilon = 0.7$   $\rho = 0.2$   $\gamma = 0.25$ . The  $R$  values are 0.99 ( $\beta = 2$ ) in (a) and 1.4 ( $\beta = 2.8$ ) in (b).

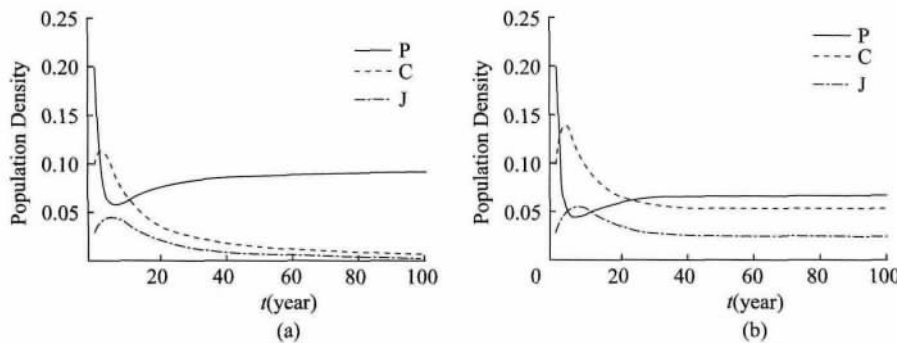


Figure 1 Time plots of the solutions of system (1) for  $R < 1$  (see (a)) and  $R > 1$  (see (b)). It illustrates that solutions converge to either the crime-free equilibrium  $E_0$  if  $R < 1$  or the crime equilibrium  $E^*$  if  $R > 1$  as  $t \rightarrow \infty$ .

The stability of  $E^*$  when  $R > 1$  is also demonstrated in the phase portrait shown in Fig. 2, which projects the solutions onto the  $(P, C)$  plane for various initial conditions. We observe that all solutions converge to  $E^*$  as  $t \rightarrow \infty$ . The parameter values are the same as in Fig. 1 (b). We have also run simulations of system (1) for a wide range of parameter values and  $E^*$  is stable in these cases.

**Result 3** Based on numerical simulations for a wide range of parameter values the crime equilibrium  $E^*$  is stable whenever it exists.

The existence and stability results of the equilibrium points of system (1) provide conditions that can be used to evaluate crime control policies and i-

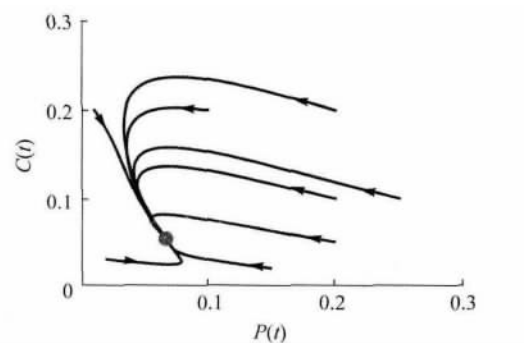


Figure 2 Phase portrait projected to the  $(P, C)$  plane for  $R > 1$ . It shows that solutions converge to  $E^*$  (marked by the solid circle) as  $t \rightarrow \infty$ .

identify cost-effective strategies as demonstrated in the next section.

### 4 Cost-effective crime control strategies

Assume that the population has stabilized at the positive equilibrium  $E^*$ , which implies the assumption that  $R > 1$  based on the analysis in the previous section. In this case the density of poverty class is

$$P^* = \frac{\mu\sigma}{(\sigma + \mu)(\gamma + \mu + \beta C^*)}$$

and the density of criminal class is  $C^*$  with  $C^* = x_+$  being the positive solution of equation (7). The parameters representing crime control and intervention are  $\gamma$  (conversion rate from  $P$  to  $R$ ) and  $\rho$  (criminal capture). Clearly the reproduction number  $R$  as well as  $P^* = P^*(\gamma, \rho)$  and  $C^* = C^*(\gamma, \rho)$  are functions of  $\gamma$  and  $\rho$ . Fig. 3 shows the dependence of  $R$  (the surface plot on the left) and the crime level  $C^*$  (the surface plot on the right) on  $\gamma$  and  $\rho$ . The parameter values used are:  $\beta = 2$ ,  $\phi = 0.1$ ,  $\mu = 0.1$ ,  $\sigma = 0.5$ ,  $\mu\delta = 0.5$ ,  $\varepsilon = 0$ . The lighter plane in the  $R$  and  $C^*$  plots corresponds to  $R = 1$  and  $C^* = 0$ , respectively. Thus for all values of  $\gamma$  and  $\rho$  such that  $R(\gamma, \rho) < 1$ , there is no equilibrium with positive  $C^*$ .

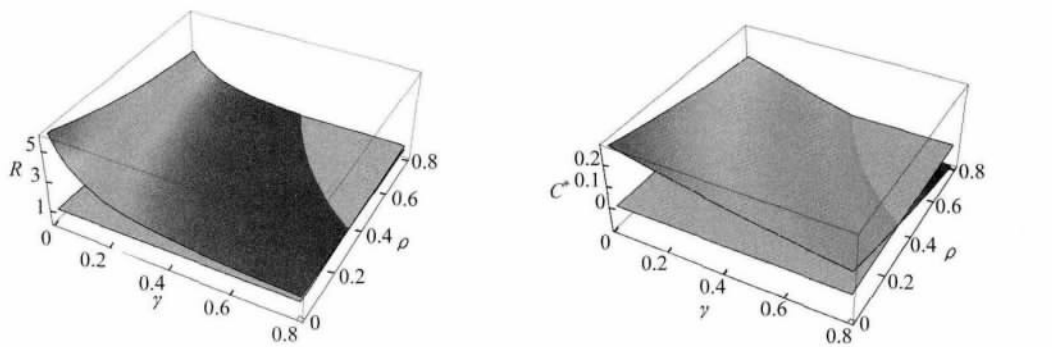


Figure 3 The 3D plots of the reproduction number  $R$  (left) and the crime level  $C^*$  (right) as a function of intervention parameters  $\gamma$  and  $\rho$ . The lighter plane represents  $R = 1$  (left) and  $C^* = 0$  (right). It illustrates that  $C^* > 0$  only for the values of  $\gamma$  and  $\rho$  such that  $R(\gamma, \rho) > 1$ .

We can explore how the control parameters  $\gamma$  and  $\rho$  may influence the crime activity by comparing  $C^*(\gamma, \rho)$  (for various  $\gamma$  and  $\rho$  values) with the crime activity  $C_0^* = C^*(\gamma_0, \rho_0)$  where  $\gamma_0$  and  $\rho_0$  represent the baseline values of control parameters (e.g. the status quo policy). For demonstration purposes consider the baseline values  $\gamma_0 = 0.4$  and  $\rho_0 = 0.1$ . Then  $C_0^* = 0.13$ . Fig. 4 is a contour plot of the crime activity  $C^*(\gamma, \rho)$  as a function of  $\gamma$  and  $\rho$ . The three contour curves correspond to the crime levels  $C^* = 0.13$ ,  $0.06$  and  $0$ . The solid circle represents the point  $(\gamma_0, \rho_0)$ , and the curve passing through the point identifies all pairs of  $(\gamma, \rho)$  that will generate the same crime activity  $C_0^*$ .

Although different combinations of  $\gamma$  and  $\rho$  can be used to obtain a given level of crime activity, the

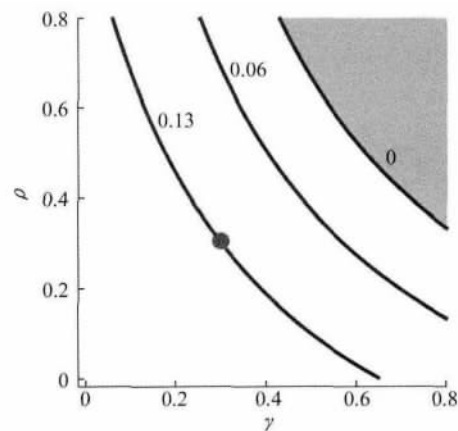


Figure 4 Contour plot of the crime level at the positive equilibrium  $C^*$  as a function of  $\gamma$  and  $\rho$ . The solid circle corresponds to the baseline values  $\gamma_0$  and  $\rho_0$  for which the criminal level is  $C_0^* = 0.13$ . The 0 level curve identifies the region (shaded) in the  $(\gamma, \rho)$  plane in which the crime level will fall to zero.

costs associated with these control measures might be very different ,some of which can be higher than the background cost  $c_0$  while others lower. Thus a better strategy would lower the cost without increasing the crime activity. There are various ways to compute the cost associated with the control measures. For the purpose of demonstration ,we consider a particular cost function. The similar approach can be used for other cost functions. Denote the total cost under the current policy  $(\gamma_0 \rho_0)$  by  $c_0$  while all other parameters are being fixed. When we vary the control parameters  $\gamma$  and  $\rho$  the total cost ,which we denote by  $c(\gamma \rho)$  ,includes the cost associated with efforts for converting poverty individuals  $(P)$  to recovered class  $(R)$  ,the cost of efforts related to capturing criminals ,as well as the cost of crimes (to the society or the victims) . An example of such function can be defined as

$$c(\gamma \rho) = w_1 B_1 C^*(\gamma \rho) + w_2 B_2 \rho C^*(\gamma \rho) + w_3 B_3 \frac{\gamma P^*(\gamma \rho)}{a + P^*(\gamma \rho)}. \tag{11}$$

$B_i$  and  $w_i (i = 1, 2, 3)$  are nonnegative constants.  $B_1$  represents the cost to the society associated with each criminal;  $B_2$  is the cost for each criminal captured; and  $B_3$  is the cost coefficient for converting a  $P$  individual to an  $R$  individual with  $a$  being a scaling constant. The use of a nonlinear function with saturation in the third term is based on the consideration that the cost will be increased at a slower rate if more people are to be helped ,as the services have already been established. The weight constants  $w_i$  will depend on the relative importance of these factors. The shape of the cost function  $c$  depends on the  $B_i$  and  $w_i$ . As these constants may not be easy to estimate ,we will present two scenarios with different sets of constants. There are few studies concerning crime-specific estimates for policy and program evaluation. It was reported that in the United States ,more than 23 million criminal offenses were committed in 2007 ,resulting in approximately \$ 15 billion in economic losses to the victims and \$ 179 billion in government expenditures on police protection ,judicial and legal activities ,and corrections<sup>[17-18]</sup>. However ,more detailed information will be needed to estimate the constants in the cost function. Here ,we have chosen the parameter values to illustrate our approach.

Fig. 5 plots the cost function given in (11). Fig. 5 (a) shows the surface of  $c(\gamma \rho)$  in relation to the baseline cost  $c_0$  (the darker plane) ,and Fig. 5 (a) plots the contour curves. The solid circle identifies the points corresponding to the baseline values  $(\gamma_0 \rho_0)$  ,and the dashed curve corresponds to the intersection of the surface  $c(\gamma \rho)$  with the  $c_0$  plane. That is ,the costs are the same for all values of  $\gamma$  and  $\rho$  along the dashed curve. The dotted curve is another level curve along which the cost has the same value and it is lower than  $c_0$ .

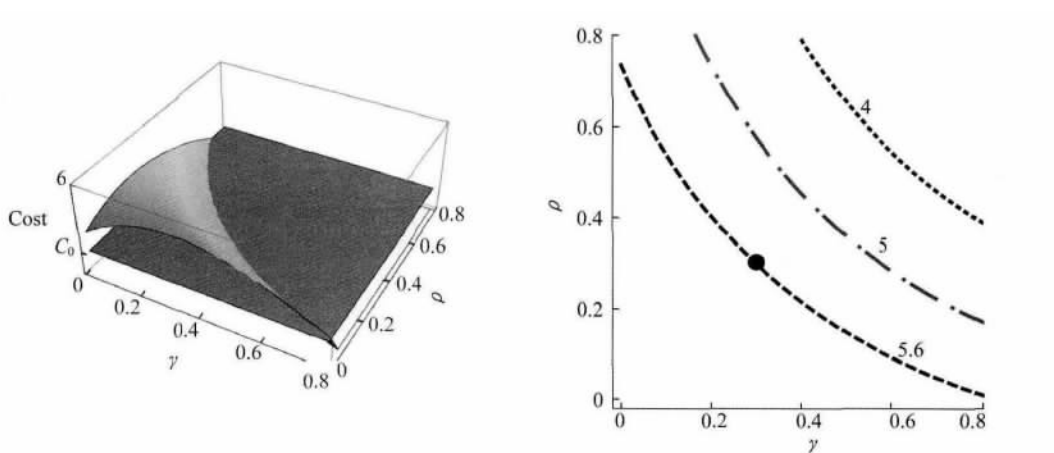


Figure 5 Cost as a function of intervention parameters  $\gamma$  and  $\rho$ . The left figure indicates the region in the  $(\gamma \rho)$  plane where the cost function  $c(\gamma \rho)$  (the lighter surface) is below the cost function  $c_0$  (the darker plane) . The figure on the right is a contour plot showing three curves. The dashed curve corresponds to  $c(\gamma \rho) = c_0 = 5.6$ . For all  $(\gamma \rho)$  above this dashed curve , $c(\gamma \rho) < c_0$ . The other two curves correspond to  $c(\gamma , \rho) = 4$  (the dotted curve) and 5 (the dotted-dash curve)

To identify new intervention strategies in comparison with current policy, we consider the following two scenarios:

**Strategy (I)** A reduced criminal level without increasing the cost i. e. ,

$$C^*(\gamma, \rho) < C_0^* \quad \text{and} \quad \mathcal{C}(\gamma, \rho) \leq \mathcal{C}_0.$$

**Strategy (II)** A reduced cost  $C$  that can lead to possible elimination of crime activity i. e. ,

$$\mathcal{C}(\gamma, \rho) < \mathcal{C}_0 \quad \text{with} \quad C^*(\gamma, \rho) \rightarrow 0.$$

To investigate Strategy (I), we examine the dependence of the crime level  $C^*$  and the total cost  $\mathcal{C}$  on the control parameters  $\gamma$  and  $\rho$  and compare these quantities with  $C_0^*$  and  $\mathcal{C}_0$ . The cost values are selected only for demonstration purposes and no specific unit is used for the total cost. When a particular population is considered with known costs of the intervention measures, more accurate parameter values can be used. Fig. 6 shows the overlap of the contour curves for crime  $C^*(\gamma, \rho)$  and for cost  $\mathcal{C}(\gamma, \rho)$ . One pair of these curves pass through the point  $(\gamma_0, \rho_0)$  (labeled by the solid circle). The cost  $\mathcal{C}$  along the dashed curve has the same value as  $\mathcal{C}_0 = 5.6$ , and the crime activity along the thicker solid curve has the value  $C^*(\gamma, \rho) = C_0^* = 0.13$ . The crime activities for  $(\gamma, \rho)$  above the solid curve are lower than  $C_0^*$ . We observe that for all points  $(\gamma, \rho)$  with  $\gamma > \gamma_0$  and  $\rho < \rho_0$  ( $\gamma < \gamma_0$  and  $\rho > \rho_0$ ), the dashed curve is above (below) the solid curve. This suggests that under the same cost  $\mathcal{C}_0$ , a strategy  $(\gamma, \rho)$  with  $\gamma > \gamma_0$  and  $\rho < \rho_0$  can lead to a reduced (an increased) crime activity  $C^* < C_0^*$  ( $C^* > C_0^*$ ). The parameter values used are:  $\beta = 2$ ,  $\phi = 0.1$ ,  $\mu = 0.1$ ,  $\sigma = 0.5$ ,  $\delta = 0.5$ ,  $\varepsilon = 0.1$ ,  $\mu_1 B_1 = 20$ ,  $\mu_2 B_2 = 20$ ,  $\mu_3 B_3 = 150$ ,  $\mu = 1$ .

For Strategy (II), we consider the zero contour curve of the crime level  $C_0^*$  (see the thin solid curve in Fig. 6). The dotted curve represents the level curve of cost  $\mathcal{C} = 4 < \mathcal{C}_0$ . Note that at the intersection of the two curves  $\gamma \approx 0.55$ . We observe that for all  $(\gamma, \rho)$  on the dotted curve with  $\gamma > 0.55$ , the level curve for  $\mathcal{C} = 4$  is above the level curve for  $C^* = 0$ , which indicates that the crime activity will fall to zero.

As mentioned earlier, the control strategies may depend on the constants  $B_i$  and  $w_i$  in the cost function (11). Variations of the relative magnitudes of these constants can lead to different strategies.

For example, in the case when  $B_1$  (cost per criminal) is small, the dependence of the total cost on  $\gamma$  and  $\rho$  is shown in Fig. 7. There are several features that are qualitatively different from the case shown in Figs. 5 and 6. Because of the lower cost of crimes ( $B_1$ ), the benefits from converting  $P$  to  $R$  (effect of  $\gamma$ ) and capturing criminal (effect of  $\rho$ ) are reduced. Thus, the total cost rises dramatically with increased  $\gamma$  and  $\rho$  (see the 3D surface on the left), which is opposite of the case shown in Fig. 5. This also affects the relative roles of  $\gamma$  and  $\rho$  when considering the reduction of crime activities without changing the baseline cost. As shown in the contour plot (right), for  $(\gamma, \rho)$  along the cost curve (dashed) with  $\gamma > \gamma_0$  and  $\rho < \rho_0$ , the cost curve is below the crime

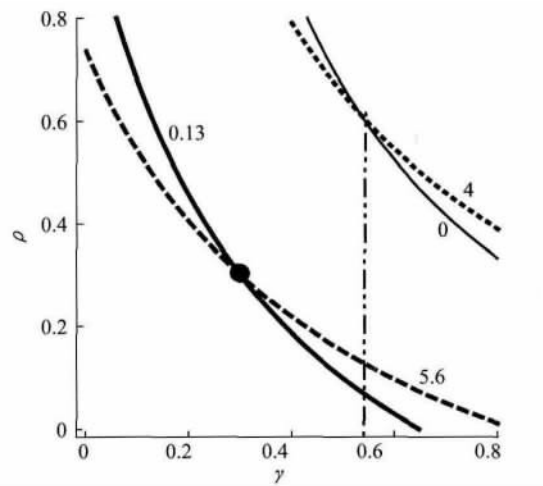


Figure 6 Overlap contour curves of the crime activity  $C^*(\gamma, \rho)$  and the cost  $\mathcal{C}(\gamma, \rho)$ . The dashed curve shows the cost with the same value as  $\mathcal{C}_0$  and the crime activity along the thicker solid curve that passes through the point  $(\gamma_0, \rho_0)$  (labeled by the solid circle) is equal to  $C_0^*$ . The dotted curve represents the cost level  $\mathcal{C} = 4$ . It indicates that for all  $(\gamma, \rho)$  on the dotted curve with  $\gamma > 0.55$ , the criminal population can be eliminated.



curve (thick solid) indicating a higher crime activity. The parameter values used are the same as in Fig. 6 except that  $w_1 B_1 = 5$ ,  $w_2 B_2 = 20$  and  $w_3 B_3 = 200$ .

## 5 Discussion

In this paper we developed a mathematical model to study the dynamics of poverty and crime. By studying the property of equilibria and their stability we derived threshold conditions which can be used to determine the prevalence and control of the crime activity. That is, the dynamics of the model depend on the reproductive number  $R$ . When  $R < 1$ , the crime level will always fall to zero, whereas when  $R > 1$ , the crime will be persistent (see Results 2, 3 and Fig. 3). Therefore, our analysis on cost-effective control is only concerned with the case  $R > 1$ . When the crime activity is persistent we explored the possibility of crime control via government interventions (represented by the parameters  $\gamma$  and  $\rho$ ) without increasing the total cost associated with the crime activity under a baseline (e. g., status quo) intervention program (represented by  $\gamma_0$  and  $\rho_0$ ). We demonstrated that under certain conditions crime control strategies can be identified (see for example, Figs. 6) if all the relevant cost and weight constants ( $B_i$  and  $w_i$ ) are known.

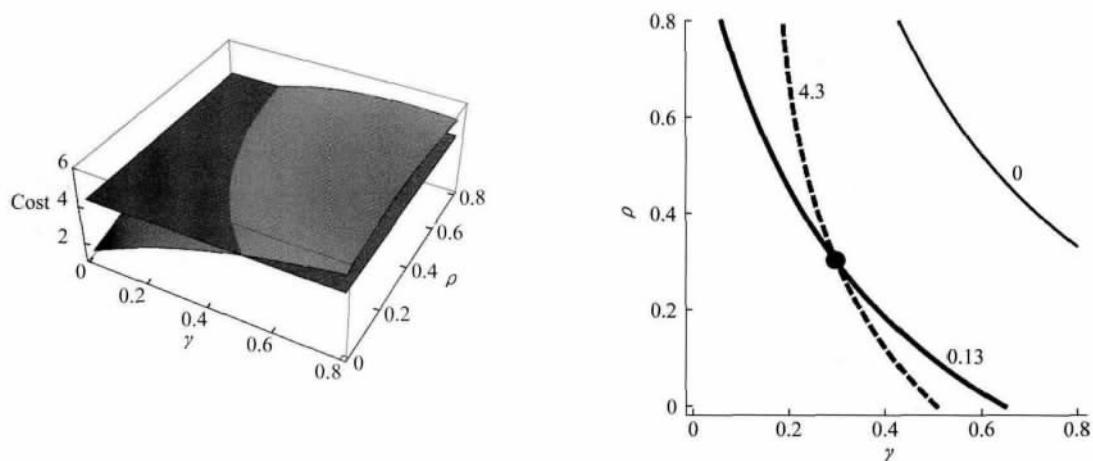


Figure 7 The 3D plot on the left is similar to Fig. 5 but for different values of  $B_i$  in the cost function. It shows that the total cost for larger  $\gamma$  and  $\rho$  is higher than  $C_0$ , which is represented by the darker plane. The contour plot on the right is similar to Fig. 6 but for different  $B_i$  values. It shows that for all  $(\gamma, \rho)$  on the dashed curve, although the total cost  $\mathcal{C}$  is the same as  $\mathcal{C}_0 = 4.3$ , the crime level  $C^*$  is higher than  $C_0^*$  if  $\gamma > \gamma_0$ . This is opposite of the case shown in Fig. 6.

We presented two examples to show how a cost-effective control strategy can be identified for a give set of parameter values. These examples illustrated that for different populations and under different conditions (determined by the baseline values  $\gamma_0$  and  $\rho_0$  as well as the cost and weight constants  $B_i$  and  $w_i$ ), the cost-effective strategies can be very different (see Figs. 6 and 7).

The implications of the model are what we expected. Naturally, eliminating all crime is not feasible. The model together with the cost function show that for a given crime level, there will be optimal values of the parameters  $\gamma$  and  $\rho$ , such that the cost of controlling crime is at a minimum. Note that we have only suggested one cost function, which is given in (11). Other forms of cost functions can be formulated depending on the particular factors associated with the population under investigation, including the poverty and crime situations, and the specific constraints for the costs of crime control and intervention, among other considerations.

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